DISTANCE METRICS FOR FACE RECOGNITION BY 2D PCA

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ABSTRACT *— Two-dimensional Principal Component Analysis (2D PCA) is a global feature extraction method for Face recognition that works upon 2D matrices rather than 1D vectors. In every Face recognition system, different distance functions used in the classification stage can yield diverse recognition rates and one of the quests for the developers is to figure out which is the most preferable function. In this paper, we concentrate on the insights of distance metrics applied for 2D PCA. A new distance metric so called weighted p, in which an exponent p and eigenvalues are used, is also proposed. To evaluate the recognition performance of those functions, comparative experiments on the face database ORL are performed. The results show that the proposed function provides 2D PCA with higher recognition rates than existing rivals.*

Keywords *— Face recognition, 2D PCA, distance functions, ORL.*

I. INTRODUCTION

Due to its diverse potential applications in reality, a plethora of systems have been proposed to solve the Face recognition (FR) problem [1] for over a quarter of a century. These systems, based on the facial features they used, can be categorized into three kinds of approach: global (also known as appearance based), local (or local feature based) and hybrid [1], [2]. In a global FR system, each face image is described as a feature vector that contains global facial features extracted from the given image. In the meantime, a local FR method relies on local features of the face obtained by using a feature extraction algorithm. A hybrid system uses both global and local features to achieve higher accuracy performance.

Among the global systems, Eigenfaces [3] and Fisherfaces [4] are the most well-known representations. In Eigenfaces, each input face image are first reshaped from its 2D form of *M rows* × *N columns* to a column vector of size $M \times N$ dimensions, then the Principal Component Analysis (PCA) is applied on a covariance matrix of training vectors to find its principal orthogonal eigenvectors for expressing every image as a linear combination of these eigenvectors. Those linear representations are used in the classification step to identify the similarities between a test image to the ones in the reference set for determining its associated identity. When the training set has more than one image per subject, Eigenfaces does not exploit this available information to enhance system accuracy as it is an unsupervised learning algorithm. Fisherfaces, on the other hand, can utilize such data to maximize extra-class variations between images of different people while minimizing the intra-class variations between those of the same person by using Linear Discriminant Analysis (LDA). On account of the fact that the intra-class variations induced by challenging factors such as changes of illumination, head pose and time-lapse always greater than the extra-class variations come from the differences of face identities [5], thus can make images of the same person extremely different, this usage of LDA is valuable and leads to better results.

In the PCA and LDA methods mentioned above, the 2D face images are transformed into 1D vectors, which are usually very high dimensional data. Consequently, the covariance matrix of these vectors is huge and the process of finding its eigenvectors and eigenvalues can be time consuming. One tactic to overcome this problem is to use Singular Value Decomposition (SVD) to compute eigenvectors and eigenvalues. Another technique is two-dimensional PCA (2D PCA) [6] in which the covariance matrix is formed directly from 2D images rather than their 1D vectors. By treating each face as a 2D image, the resulting covariance matrix is smaller and eigenvectors are more accurate since the computational process does not require so many operations as in Eigenfaces. In [7], the author introduced the idea of complete 2D-PCA by computing two covariance matrix and projected each 2D input matrix in two directions to obtain smaller projected matrix in both row and column. The complete 2D-PCA did not achieved higher result than original 2D-PCA but reduce the needed coefficients. Another extension of 2D-PCA was presented in [8] and the method gave some improvements in recognition rates.

There are several stages within a 2D PCA based FR system, they are face detection, face alignment, illumination normalization, feature extraction, dimensionality reduction and classification. In this paper, we work with images from ORL database and for making our results comparable with those of other systems, we do not apply any algorithm for the first three stages. As 2D PCA is an appearance based technique, it is used for both feature extraction and dimensionality reduction purposes. In the classification stage the k-NN classifier and a distance function are used to determine the identity of a test image. According to [9], different distance measures provide different accuracy performance for a PCA system. Learning from that, our motivation with this paper is finding the best fit distance metric for 2D PCA. Towards this end, we first explore different distance functions and then propose a new measure method for using with 2D PCA. Experimental results upon ORL database and comparisons with state-of-the-art systems show that our proposed distance function offers the highest recognition rate.

The rest of this paper is structured as follows. The next section presents the details of 2D PCA and all the distance metrics used with it. The experimental results and comparisons with other methods are given in Section III. Section IV provides the conclusions and perspective of the paper.

II. DISTANCE METRICS FOR 2D PCA

A. 2D PCA overview

From **K** training images, which are represented as 2D N x M matrices $\mathbf{X_i}$ (i=1..K), their covariance matrix **G** is computed as:

$$\boldsymbol{G} = \frac{\boldsymbol{1}}{\boldsymbol{K}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{K}}(\boldsymbol{X_i} - \overline{\boldsymbol{X}})^{\boldsymbol{T}}(\boldsymbol{X_i} - \overline{\boldsymbol{X}}), \tag{1}$$

where $\bar{X}$ is the mean matrix of all X_i:

$$\overline{\boldsymbol{X}} = \frac{\boldsymbol{1}}{\boldsymbol{K}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{K}}\boldsymbol{X_i}. \tag{2}$$

From equation 1 one can notice that **G** will have size of N x N and is a symmetric matrix. Next **N** eigenvectors $\mathbf{u_i}$ and their N associated eigenvalues $\boldsymbol{\lambda_i}$ will be calculated by using eigenvalue decomposition method upon matrix **G** as:

$$[\boldsymbol{U}, \boldsymbol{\Lambda}] = \boldsymbol{eig}(\boldsymbol{G}). \tag{3}$$

Then, all $\mathbf{u_i}$ are sorted in descending order based on their associated eigenvalues $\boldsymbol{\lambda_i}$ (i=1..K). The projection matrix $\mathbf{U_{proj}}$ is constituted from only first **d** eigenvectors corresponding to **d** largest eigenvalues:

$$\boldsymbol{U_{proj}} = [\boldsymbol{u_1}\boldsymbol{u_2} \ldots \boldsymbol{u_d}]. \tag{4}$$

Each gallery image **Y** is next projected into 2D PCA subspace as:

$$\widetilde{\boldsymbol{Y}} = (\boldsymbol{Y} - \overline{\boldsymbol{X}})\boldsymbol{U_{proj}}. \tag{5}$$

When a test image **Z** comes, it is also projected by:

$$\widetilde{\boldsymbol{Z}} = (\boldsymbol{Z} - \overline{\boldsymbol{X}})\boldsymbol{U_{proj}}. \tag{6}$$

The results obtained after the projection step above are 2D matrices of size N x d, which will be fed into the classification step to recognize their labels. In this work, by empirically selection, we fix d = 4 for all experiments on ORL face database.

To determine the identity of image **Z**, k-NN (in this work, we set k = 1) classifier is applied with a distance function to find the most similar image in the gallery set and assigns its identity to **Z**.

B. Distance metrics for 2D PCA

1. Currently used distance metrics

In this paper, we assess the accuracy performance of different distance metrics when applying with 2D PCA. Let **X** and **Y** be the two projected matrix of size **R x C** obtained from 2D PCA, their distance is computed by the following functions:

Euclidean distance:

$$\boldsymbol{d}(\boldsymbol{X}, \boldsymbol{Y}) = \sqrt{\sum_{\boldsymbol{i=1}}^{\boldsymbol{R}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{C}}(\boldsymbol{x_{i,j}} - \boldsymbol{y_{i,j}})^{\boldsymbol{2}}}\,. \tag{7}$$

Cosine angle distance:

$$\boldsymbol{d}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{-\sum_{\boldsymbol{i=1}}^{\boldsymbol{R}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{C}}\boldsymbol{x_{i,j}}\boldsymbol{y_{i,j}}}{\sqrt{\sum_{\boldsymbol{i=1}}^{\boldsymbol{R}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{C}}\boldsymbol{x_{i,j}^2}\sum_{\boldsymbol{i=1}}^{\boldsymbol{R}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{C}}\boldsymbol{y_{i,j}^2}}} \tag{8}$$

Mean square error (MSE) distance:

$$\boldsymbol{d}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{\boldsymbol{1}}{\boldsymbol{RC}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{R}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{C}}(\boldsymbol{x_{i,j}} - \boldsymbol{y_{i,j}})^{\boldsymbol{2}}. \tag{9}$$

Manhattan distance:

$$d(X,Y) = \sum_{i=1}^{R}\sum_{j=1}^{C}\left|x_{i,j} - y_{i,j}\right|. \tag{10}$$

Correlation distance:

$$d(X,Y) = \frac{-\left(R * C * \sum_{i=1}^{R}\sum_{j=1}^{C} x_{i,j}y_{i,j} - \sum_{i=1}^{R}\sum_{j=1}^{C} x_{i,j}\sum_{i=1}^{R}\sum_{j=1}^{C} y_{i,j}\right)}{\sqrt{\left((R * C * \sum_{i=1}^{R}\sum_{j=1}^{C} x_{i,j}^2 - (\sum_{i=1}^{R}\sum_{j=1}^{C} x_{i,j})^2\right) * \left(R * C * \sum_{i=1}^{R}\sum_{j=1}^{C} y_{i,j}^2 - (\sum_{i=1}^{R}\sum_{j=1}^{C} y_{i,j})^2\right)}}. \tag{11}$$

Modified Manhattan distance:

$$d(X,Y) = \frac{\sum_{i=1}^{R}\sum_{j=1}^{C}\left|x_{i,j} - y_{i,j}\right|}{\sum_{i=1}^{R}\sum_{j=1}^{C} x_{i,j}^2 \sum_{i=1}^{R}\sum_{j=1}^{C} y_{i,j}^2}. \tag{12}$$

2. Weighted p exponent, the proposed distance metric for 2D PCA

Aside from aforementioned distance functions in the previous section, we propose a new distance measure for using with 2D PCA. While all the above function do not use eigenvalues, we employ them to weight each column vector of the projected matrix by the corresponding eigenvalue. An exponent **p** is also used as follow:

$$d(X,Y) = \sqrt[p]{\sum_{i=1}^{R}\left(\sum_{j=1}^{C}\frac{\left|x_{i,j} - y_{i,j}\right|^p}{\lambda_j}\right)}, \tag{13}$$

where λ_j is the jth eigenvalue computed from 2D PCA. Via empirically experiments, p is assigned to be 2.5 for all tests carried out in this paper.

III. EXPERIMENTAL RESULTS

A. ORL database

In order to evaluate the proposed distance function and figure out which is the best fit distance metric for 2D PCA, we employ the Olivetti and Oracle Laboratory (ORL, which can be downloaded from address http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html) face database to carry out experiments and compare our results with those of state-of-the-art systems. The database contains 400 grayscale images (same size of 92 x 112 resolution) of 40 distinct people (every person has 10 images, for more details, see Figure 1) taken under variations of illumination, facial expressions and time-lapse. We select first k images (k=1, 2, 3, 4 and 5) of each person for being in the training set, the rest are for the test set. Thus, we have 5 different experiments in which each one is tested with different distance measures and the yielded results are compared with those of other existing systems.

Figure 1. Sample images from ORL database

B. Experimental results

The comparison results between different distance functions, including our proposed one, are shown in table 1. It can be clearly seen that our proposed distance function has highest accuracy performance in all experiments associated to different numbers of training images. Among other distance functions, the modified Manhattan distance yields the best results while the cosine distance provides the lowest recognition rates. Another conclusion deduced from table 1 is that, except the results brought by cosine distance, the more images we use for the training stage, the higher recognition performance we obtain.

Table 1. 1-rank (%) comparison between different distance metrics when being used with 2D PCA

Distance metric	**K**				
	1	**2**	**3**	**4**	**5**
Cosine	33.1	29.7	26.8	27.1	26.5
Euclidean	72.8	84.1	84.6	88.3	89.0
MSE	72.8	84.1	84.6	88.3	89.0
Manhattan	75.3	85.9	87.5	89.2	91.0
Correlation	75.6	86.9	88.2	92.1	92.0
Modified Manhattan	75.6	87.5	88.6	90.4	94.0
Weighted p	**76.7**	**88.4**	**88.9**	**93.3**	**96.0**

In table 2, the comparisons between our best results offered by weighted p distance and that of other existing systems are shown. One can observe that our results are higher or comparative in compared with state-of-the-art results. The most considerable result is obtained when the number of training images for each person is 5 (in the last column), this is also the most widely evaluation performed with the ORL database.

Table 2. 1-rank comparison of 2D PCA using weighted p distance with other existing systems

Method	**K**				
	1	**2**	**3**	**4**	**5**
PCA	75.3	84.4	85.0	88.8	91.0
Complete 2D PCA [7]	78.9	87.5	90.7	91.7	91.5
E2DPCA [8]	N/A[1]				93.0
2D-LDA [10]					94.0
2D-FPCA [11]	N/A[1]	89.1	90.0	93.8	95.5
Our	**76.7**	**88.4**	**88.9**	**93.3**	**96.0**

IV. CONCLUSION

An extensive evaluation of different distance metrics using in 2D PCA has been presented in this paper. By exploring several available distance functions and proposing a new method to measure projected matrices in the classification stage of a 2D PCA based FR system, we have shown that the accuracy performance of the system can be varied due to the distance metric it uses and the weighted p distance is a good candidate for being used with 2D PCA when it achieves better results than other rivals.

V. REFERENCES

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TÓM TẮT *- Phân tích thành phần chính 2 chiều (2D PCA) là một phương pháp trích chọn đặt điểm toàn cục trong nhận dạng mặt làm việc trực tiếp với các ma trận 2 chiều thay vì các vectơ 1 chiều. Trong tất cả các hệ thống nhận dạng mặt người, các hàm khoảng cách khác nhau được sử dụng trong pha phân lớp sẽ cho các tỉ lệ nhận dạng khác nhau và một trong các nhiệm vụ của nhà phát triển là phải tìm xem đâu là hàm khoảng cách thích hợp nhất. Trong bài báo này, chúng tôi tập trung vào các hàm khoảng cách được áp dụng cho phương pháp 2D PCA. Một hàm khoảng cách mới được gọi là trọng số p, trong đó sử dụng số mũ p và các giá trị riêng, cũng được đề xuất. Để đánh giá hiệu năng nhận dạng của các hàm khoảng cách này, các thử nghiệm so sánh trên cơ sở dữ liệu ảnh mặt ORL đã được thực hiện. Kết quả cho thấy hàm khoảng cách mà chúng tôi đề xuất có tỉ lệ nhận dạng cao hơn so với các hàm khoảng cách khác.*

Từ khóa *- Nhận dạng mặt người, phương pháp 2D PCA, hàm khoảng cách cho 2D PCA, cơ sở dữ liệu ORL.*