

# SOME SELECTED PROBLEMS OF MODERN SOFT COMPUTING

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## I. INTRODUCTION

Soft computing differs from conventional (hard) computing in that, unlike hard computing, it is Tolerant of imprecision, uncertainty, partial truth and approximation. In effect, the role model for

Soft computing is the human mind.

Components of Soft computing include:

- Fuzzy Logic
- Neural networks
- Evolutionary computation
- Support vector Machines

In this paper, we shall present some selected problems of modern soft computing based on the new concept "Picture Fuzzy sets" (PFS, 2013). Some new basic definitions, operators and relative basic propositions of Picture uzzzy Theory should be presented.

We shall also discuss on the main operations of Picture Fuzzy Logic and shall present some new results of the research group of Dr. Le Hoang Son on Picture Fuzzy Clustering method with some applications

## II. PART 1. PICTURE FUZZY SET THEORY – A NEW DEVELOPMENT OF FUZZY SET THEORY

In this section we propose some definitions of picture fuzzy sets and some operators on PFS.

**Definition 1.1.** A picture fuzzy set  $A$  on a universe  $X$  is an object of the form

$$A = \{ (x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X \},$$

where  $\mu_A(x) \in [0,1]$  is called the "degree of positive membership of  $x$  in  $A$ ",  $\eta_A(x) \in [0,1]$  is called the "degree of neutral membership of  $x$  in  $A$ " and  $\nu_A(x) \in [0,1]$  is called the "degree of negative membership of  $x$  in  $A$ ", and where  $\mu_A$ ,  $\eta_A$  and  $\nu_A$  satisfy the following condition:

$$(\forall x \in X) \quad (\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1) .$$

Now  $(1 - (\mu_A(x) + \eta_A(x) + \nu_A(x)))$  could be called the "degree of refusal membership" of  $x$  in  $A$  Let  $PFS(X)$  denote the set of all the picture fuzzy sets on a universe  $X$ . Basically, picture fuzzy sets based models may be adequate in situations when we face human opinions involving more answers of types: yes, abstain, no, refusal.

Voting can be a good example of such a situation as the human voters may be divided into four groups of those who: vote for, abstain, vote against, the refusal of the voting.

PFS is a direct generalization of the fuzzy set was introduced by Zadeh 1965 [1] and the intuitionistic fuzzy set was proposed by Atanassov 1983 [3].

In this paper, let  $IFS(X)$  denote the set of all the intuitionistic fuzzy set IFSs on a universe  $X$ .

**Definition 1.2.** For every two PFSs  $A$  and  $B$ , the union, intersection and complement are defined as follows:

- $A \subseteq B$  iff  $(\forall x \in X, \mu_A(x) \leq \mu_B(x)$  and  $\eta_A(x) \leq \eta_B(x)$  and  $\nu_A(x) \geq \nu_B(x))$  ;

- $A = B$  iff  $(A \subseteq B \text{ and } B \subseteq A)$  ;
- $A \cup B = \{(x, (\max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x), \min(\nu_A(x), \nu_B(x))) | x \in X)\}$  ;
- $A \cap B = \{(x, (\min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x), \max(\nu_A(x), \nu_B(x))) | x \in X)\}$  ;
- $co(A) = \bar{A} = \{(\nu_A(x), \eta_A(x), \mu_A(x)) | x \in X\}$  .

Now we will propose a generalization of interval-valued fuzzy set  $A$  . Here the  $\text{int}([0, 1])$  stands for the set of all closed subinterval of  $[0, 1]$  .

**Definition 1.3.** Let  $[a_1, b_1], [a_2, b_2] \in \text{int}([0, 1])$  . We define

$$[a_1, b_1] \leq [a_2, b_2], \text{ iff } a_1 \leq a_2, b_1 \leq b_2; \quad [a_1, b_1] \lesssim [a_2, b_2] \text{ iff } a_1 \leq a_2, b_1 \geq b_2;$$

$$[a_1, b_1] = [a_2, b_2], \text{ iff } a_1 = a_2, b_1 = b_2.$$

**Definition 1.4.** An interval-valued picture fuzzy set  $A$  on a universe  $X$  (IvPFS, in short) is an object of the form

$$A = \{(x, M_A(x), L_A(x), N_A(x)) | x \in X\},$$

where  $M_A : X \rightarrow \text{int}([0, 1])$  ,  $M_A(x) = [M_{AL}(x), M_{AU}(x)] \in \text{int}([0, 1])$ ,

$$L_A : X \rightarrow \text{int}([0, 1])$$
 ,  $L_A(x) = [L_{AL}(x), L_{AU}(x)] \in \text{int}([0, 1])$  ,

$$N_A : X \rightarrow \text{int}([0, 1])$$
 ,  $N_A(x) = [N_{AL}(x), N_{AU}(x)] \in \text{int}([0, 1])$ ,

satisfy the following condition:

$$(\forall x \in X) \quad (\sup M_A(x) + \sup L_A(x) + \sup N_A(x) \leq 1) \text{ .}$$

Let  $IvPFS(X)$  denote the set of all the interval-valued picture fuzzy set IvPFSs on a universe  $X$  .

**Definition 1.5.** For every two IvPFSs  $A$  and  $B$ , the inclusion, union, intersection and complement are defined as follows:

- $A \subseteq B$  iff  $(\forall x \in X)(M_{AL}(x) \leq M_{BL}(x) \text{ and } M_{AU}(x) \leq M_{BU}(x)) \text{ and } (L_{AL}(x) \leq L_{BL}(x) \text{ and } L_{AU}(x) \leq L_{BU}(x)) \text{ and } (N_{AL}(x) \geq N_{BL}(x) \text{ and } N_{AU}(x) \geq N_{BU}(x))$ .
- $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$  ,
- $A \cup B = \{(x, ([M_{AL}(x) \vee M_{BL}(x), M_{AU}(x) \vee M_{BU}(x)], [L_{AL}(x) \wedge L_{BL}(x), L_{AU}(x) \wedge L_{BU}(x)], [N_{AL}(x) \wedge N_{BL}(x), N_{AU}(x) \wedge N_{BU}(x)]) | x \in X\}$
- $A \cap B = \{(x, ([M_{AL}(x) \wedge M_{BL}(x), M_{AU}(x) \wedge M_{BU}(x)], [L_{AL}(x) \wedge L_{BL}(x), L_{AU}(x) \wedge L_{BU}(x)], [N_{AL}(x) \vee N_{BL}(x), N_{AU}(x) \vee N_{BU}(x)]) | x \in X\}$

where  $\vee$  and  $\wedge$  stand for max and min operators respectively

- $coA = \bar{A} = \{(x, N_A(x), L_A(x), M_A(x)) | x \in X\}$  .

**Definition 1.6.** Let  $X_1$  and  $X_2$  be two universes and let  $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X_1\}$  and  $B = \{(y, \mu_B(y), \eta_B(y), \nu_B(y)) | y \in X_2\}$  be two PFSs. We define the Cartesian product of these two PFS's

- $A \times_1 B = \{((x, y), \mu_A(x), \mu_B(y), \eta_A(x), \eta_B(y), \nu_A(x), \nu_B(y)) | x \in X_1, y \in X_2\}$ ;

$$\bullet A \times_2 B = \{((x, y), \mu_A(x) \wedge \mu_B(y), \eta_A(x) \wedge \eta_B(y), \nu_A(x) \vee \nu_B(y)) \mid x \in X_1, y \in X_2\}.$$

Basic propositions on PFSs and IvPFSs could be seen in [7,8].

**c. Zadeh’ Extension Principle for PFS**

The Zadeh’ Extension Principle is an important tool for many problems of Fuzzy Set Theory, Fuzzy Control and applications. Now a version of the Zadeh’ Extension Principle for PFS is presented. We need the following simple proposition.

**Definition 2.1.** Let  $X_1$  and  $X_2$  be two universums and let  $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X_1\}$  and  $B = \{(y, \mu_B(y), \eta_B(y), \nu_B(y)) \mid y \in X_2\}$  be two PFSs.

We define the Cartesian product of these two PFS’s

$$A \times_2 B = \left\{ \left( (x, y), \mu_A(x) \wedge \mu_B(y), \eta_A(x) \wedge \eta_B(y), \nu_A(x) \vee \nu_B(y) \right) \mid x \in X_1, y \in X_2 \right\}.$$

**Proposition 2.2.** Let for  $i=1, \dots, n$ ,  $X_i$  be a universum and  $A_i = \{(x, \mu_{A_i}(x), \eta_{A_i}(x), \nu_{A_i}(x)) \mid x \in X_i\}$  be a PFS on  $X_i$ . Then the Cartesian product of PFS’s

$$B^n = \prod_{i=1}^n A_i = \left\{ \left( x, \bigwedge_{i=1}^n \mu_{A_i}(x_i), \bigwedge_{i=1}^n \eta_{A_i}(x_i), \bigvee_{i=1}^n \nu_{A_i}(x_i) \right) \mid \forall x_i \in X_i, i = 1, \dots, n, x = (x_1, \dots, x_n) \right\}.$$

is a PFS on  $\prod_{i=1}^n X_i$ .

**Proposition 2.3. The Zadeh’Extension Principle for PFS.**

Let for  $i=1, 2, \dots, n$ ,  $U_i$  be a universum and let  $V \neq \emptyset$ . Let  $f : \prod_{i=1}^n U_i \rightarrow V$  be a mapping, where  $y = f(z_1, \dots, z_n)$ . Let  $z_i$  is a linguistic variable on  $U_i$  for  $i=1, \dots, n$ . Suppose for all  $i$ , where  $z_i$  is  $A_i$  and  $A_i$  is a PFS on  $U_i$ , then the output of the mapping  $f$  is  $B$ ,  $B$  is a PFS on  $V$  defined for  $y \in V$  by

$$B(y) = \begin{cases} \left( \bigvee_{D(y)} \left( \bigwedge_{i=1}^n x_{1i} \right), \bigwedge_{D(y)} \left( \bigwedge_{i=1}^n x_{2i} \right), \bigwedge_{D(y)} \left( \bigvee_{i=1}^n x_{3i} \right) \right) & \text{if } f^{-1}(y) \neq \emptyset \\ (0, 0, 0) & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

where

$$x_{1i} = \mu_{A_i}(u_i), x_{2i} = \eta_{A_i}(u_i), x_{3i} = \nu_{A_i}(u_i), \\ D(y) = f^{-1}(y) = \{u = (u_1, \dots, u_n) : f(u) = y\}$$

**III. PART 2. SOME PICTURE FUZZY LOGIC OPERATORS**

Consider the set  $D^* = \{x = (x_1, x_2, x_3) \mid x \in [0, 1]^3, x_1 + x_2 + x_3 \leq 1\}$ . From now on, we will assume that if  $x \in D^*$ , then  $x_1, x_2$  and  $x_3$  denote, respectively, the first, the second and the third component of  $x$ , i.e.,  $x = (x_1, x_2, x_3)$ . We have a complete lattice  $(D^*, \leq_1)$  defined by

$$\forall x, y \in D^* : x \leq_1 y \Leftrightarrow \{x_1 < y_1, x_3 \geq y_3\} \vee \{x_1 = y_1, x_3 > y_3\} \vee \{x_1 = y_1, x_3 = y_3, x_2 \leq y_2\}, \\ x = y \Leftrightarrow \{x_1 = y_1, x_3 = y_3, x_2 = y_2\}.$$

Denote for each  $x \in D^*$ ,  $I(x) = \{y = (x_1, y_2, x_3) \in D^* : y_2 \leq 1 - x_1 - x_3\}$ .

**a. Picture fuzzy negations**

**Definition 3.1.** A picture fuzzy negation  $N$  is a nonincreasing function:  $N : D^* \rightarrow D^*$  satisfies:

$$N(0_{D^*}) = 1_{D^*}, N(1_{D^*}) = 0_{D^*}$$

A picture fuzzy negation  $N$  is called an involution iff  $N$  satisfies  $N(N(x)) = x, \forall x \in D^*$ .

For example:  $N_S(x) = (x_3, x_4, x_1), \forall x \in D^*$ , is an involutive picture fuzzy negation, called standard picture fuzzy negation, where  $x_4 = 1 - (x_1 + x_2 + x_3), \forall x \in D^*$ .

**b. Picture fuzzy t-norms & Picture fuzzy t-conorms**

**Definition 3.2** A mapping  $T : D^* \times D^* \rightarrow D^*$  is a picture fuzzy t-norm if  $T$  satisfies the following conditions:

1.  $T(x, y) = T(y, x), \forall x, y \in D^*$ .
2.  $T(x, T(y, z)) = T(T(x, y), z), \forall x, y, z \in D^*$ .
3.  $T(x, y) \leq_1 T(x, z), \forall x, y, z \in D^*, y \leq_1 z$ .
4.  $T(x, 1_{D^*}) \in I(x), \forall x \in D^*$ .

**Definition 3.3** A mapping  $S : D^* \times D^* \rightarrow D^*$  is a picture fuzzy t-conorm if  $S$  satisfies satisfies the following conditions:

1.  $S(x, y) = S(y, x), \forall x, y \in D^*$ .
2.  $S(x, S(y, z)) = S(S(x, y), z), \forall x, y, z \in D^*$ .
3.  $S(x, y) \leq_1 S(x, z), \forall x, y, z \in D^*, y \leq_1 z$ .
4.  $S(x, 0_{D^*}) \in I(x), \forall x \in D^*$ .

**Some examples:** Some picture fuzzy t-norms, for all  $x, y \in D^*$  :

1.  $T_0(x, y) = (\min(x_1, y_1), 0, \max(x_3, y_3))$ .
2.  $T_{03}(x, y) = (x_1 y_1, 0, \max(x_3, y_3))$ .
3.  $T_{06}(x, y) = (\max(0, x_1 + y_1 - 1), 0, \min(1, x_3 + y_3))$ .

Some picture fuzzy t-conorms, for all  $x, y \in D^*$  :

1.  $S_0(x, y) = (\max(x_1, y_1), 0, \min(x_3, y_3))$ .
2.  $S_{03}(x, y) = (\max(x_1, y_1), 0, x_3 y_3)$ .
3.  $S_{04}(x, y) = (x_1 + y_1 - x_1 y_1, 0, x_3 y_3)$ .

**Definition 3.4** A picture fuzzy t-norm  $T$  is called t-representable iff there exist two t-norms  $t_1, t_2$  and a t-conorm  $S_3$  on  $[0,1]$  satisfy:

$$T(x, y) = (t_1(x_1, y_1), t_2(x_2, y_2), s_3(x_3, y_3)), \forall x, y \in D^*.$$

**Definition 3.5** A picture fuzzy t-conorm  $S$  is called t-representable iff there exist two t-norms  $t_1, t_2$  and a t-conorm  $s_3$  on  $[0, 1]$  satisfy:

$$S(x, y) = (s_3(x_1, y_1), t_2(x_2, y_2), t_1(x_3, y_3)), \forall x, y \in D^*.$$

**Some examples:** Some t-representable picture fuzzy t-norms, for all  $x, y \in D^*$  :

$$4. T_{\min}(x, y) = (\min(x_1, y_1), \min(x_2, y_2), \max(x_3, y_3)).$$

$$5. T_2(x, y) = (\min(x_1, y_1), x_2 y_2, \max(x_3, y_3)).$$

$$6. T_3(x, y) = (x_1 y_1, x_2 y_2, \max(x_3, y_3)).$$

$$7. T_6(x, y) = (\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3)).$$

**Some examples:** Some t-representable picture fuzzy t-conorms, for all  $x, y \in D^*$  :

$$4. S_{\max}(x, y) = (\max(x_1, y_1), \min(x_2, y_2), \min(x_3, y_3)).$$

$$5. S_2(x, y) = (\max(x_1, y_1), x_2 y_2, \min(x_3, y_3)).$$

$$6. S_3(x, y) = (\max(x_1, y_1), x_2 y_2, x_3 y_3).$$

$$7. S_4(x, y) = (x_1 + y_1 - x_1 y_1, x_2 y_2, x_3 y_3).$$

$$8. S_5(x, y) = \left( x_1 \vee y_1, \begin{cases} x_2 \wedge y_2 & \text{if } x_2 \vee y_2 = 1 \\ 0 & \text{if } x_2 \vee y_2 < 1 \end{cases}, x_3 \wedge y_3 \right).$$

Some new results on picture fuzzy logic operators were given in [9]. Some basic proposition on Picture fuzzy soft sets were given in [10].

#### IV. PART 3. PICTURE FUZZY CLUSTERING METHOD WITH APPLICATIONS

##### a. Picture Picture Fuzzy Clustering Method

In this section, a picture fuzzy model for clustering problem is given. Supposing that there is a dataset  $X$  consisting of  $N$  data points in  $d$  dimensions. Let us divide the dataset into  $C$  groups satisfying the objective function below.

$$J = \sum_{k=1}^N \sum_{j=1}^C (u_{kj} (2 - \xi_{kj}))^m \|X_k - V_j\|^2 + \sum_{k=1}^N \sum_{j=1}^C \eta_{kj} (\log \eta_{kj} + \xi_{kj}) \rightarrow \min, \quad (22)$$

Some constraints are defined as follows.

$$u_{kj} + \eta_{kj} + \xi_{kj} \leq 1, \quad (23)$$

$$\sum_{j=1}^C (u_{kj} (2 - \xi_{kj})) = 1, \quad (24)$$

$$\sum_{j=1}^C \left( \eta_{kj} + \frac{\xi_{kj}}{C} \right) = 1, \quad (25)$$

$$k = 1, \dots, N, j = 1, \dots, C.$$

Now, Lagrangian method is used to determine the optimal solutions of the model.

**Theorem 1.** The optimal solutions of the systems (22-25) are:

$$\xi_{kj} = 1 - (u_{kj} + \eta_{kj}) - \left(1 - (u_{kj} + \eta_{kj})^\alpha\right)^{\frac{1}{\alpha}}, \quad (k = 1, \dots, N, j = 1, \dots, C), \tag{26}$$

$$u_{kj} = \frac{1}{\sum_{i=1}^C (2 - \xi_{ki}) \left(\frac{\|X_k - V_j\|}{\|X_k - V_i\|}\right)^{\frac{2}{m-1}}}, \quad (k = 1, \dots, N, j = 1, \dots, C), \tag{27}$$

$$\eta_{kj} = \frac{e^{-\xi_{kj}}}{\sum_{i=1}^C e^{-\xi_{ki}}} \left(1 - \frac{1}{C} \sum_{i=1}^C \xi_{ki}\right), \quad (k = 1, \dots, N, j = 1, \dots, C), \tag{28}$$

$$V_j = \frac{\sum_{k=1}^N (u_{kj} (2 - \xi_{kj}))^m X_k}{\sum_{k=1}^N (u_{kj} (2 - \xi_{kj}))^m}, \quad (j = 1, \dots, C). \tag{29}$$

**Fuzzy Clustering Method on Picture Fuzzy Sets**

- I:** Data  $X$  whose number of elements ( $N$ ) in  $d$  dimensions; Number of clusters ( $C$ ); threshold  $\varepsilon$ ; fuzzifier  $m$ ; exponent  $\alpha$  and the maximal number of iteration  $\max Steps > 0$
- O:** Matrices  $u, \eta, \xi$  and centers  $V$ ;

**FC-PFS Algorithm:**

- 1:  $t = 0$
- 2:  $u_{kj}^{(t)} \leftarrow random; \eta_{kj}^{(t)} \leftarrow random; \xi_{kj}^{(t)} \leftarrow random (k = \overline{1, N}, j = \overline{1, C})$  satisfying the sum-row constraint
- 3: Repeat
- 4:  $t = t + 1$
- 5: Calculate  $V_j^{(t)} (j = \overline{1, C})$  by equation (29)
- 6: Calculate  $u_{kj}^{(t)} (k = \overline{1, N}; j = \overline{1, C})$  by equation (27)
- 7: Calculate  $\eta_{kj}^{(t)} (k = \overline{1, N}; j = \overline{1, C})$  by equation (28)
- 8: Calculate  $\xi_{kj}^{(t)} (k = \overline{1, N}; j = \overline{1, C})$  by equation (26)
- 9: Until  $\|u^{(t)} - u^{(t-1)}\| + \|\eta^{(t)} - \eta^{(t-1)}\| + \|\xi^{(t)} - \xi^{(t-1)}\| \leq \varepsilon$  or  $t > \max Steps$

**b.** The distributed and automatic versions of FC-PFS namely DPFCM and AFC-PFS are presented in [12,13], respectively.

Many new applications of the new approach to soft computing problems were given in [11,12,13,14].

**V. SOME OPEN PROBLEMS**

**5.1 Problem 1. Subclasses of De Morgan operators triples.**

In the fuzzy operator theory many interesting De Morgan operators triples were studied and presented in [2,4,5,6]. These computing systems are very important in the algorithms for many practical problems with the fuzzy information and they are also interesting for theoretical researches.

What De Morgan picture operators triple subclasses are important and interesting for intuitionistic fuzzy information and for picture fuzzy information ?

### 5.2 Problem 2. What fuzzy implication operators are important for picture fuzzy systems ?

For these questions, you can see the literature on the Fuzzy Systems and the Intuitionistic Fuzzy Systems.

We obtained only some first results.

### 5.3. Problem 3. Applications to Computational Intelligence problems

Dr. Le Hoang Son et al. in [11,13,14] has shown that “Picture Fuzzy Sets Theory” is a new approach to many Computational Intelligence problems .

This paper reviewed some recent researches about soft computing methods on picture fuzzy sets. Each method has been validated on the equivalent paper. They contribute an important role to the development of advanced fuzzy sets and soft computing fields. Further works of this direction can be lean in the following ways:

- Picture Neuro-Fuzzy Systems.
- Picture Recommender Systems.
- Applications on Medical Science and other fields.

We hope that there are many Soft computing problems in uncertainty environment could be solve their questions with the new concept and the new tools of picture fuzzy logic and picture fuzzy systems

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