# **Position control and vibration suppression of a flexible overhead crane system**

*Điều khiển vị trí và dập dao động tải trọng cho hệ thống nâng hạ*

**T. L. Nguyen\*,\*\* and M. D. Duong\* \*Department of Industrial Automation \*\* Institute for Control Engineering and Automation Hanoi University of Science and Technology (E-mail: lam.nguyentung@hust.edu.vn, duc.duongminh@hust.edu.vn)**

#### **Abstract:**

Trolley positioning and payload swinging control problem of a flexible overhead crane system is addressed in this paper. The system's equations of motion that couple the crane's cable and actuators dynamics are derived via extended Hamilton's principle. The control signal is designed based on the Lyapunov direct method and backstepping control. The stability of the closed loop system is proven. Numerical simulations are included to demonstrate the effectiveness of the proposed control.

**Keywords:** Flexible systems, overhead crane, field oriented control, Lyapunov direct method.

## **Tóm tắt**

Bài báo đề cập đến vấn đề điều khiển vị trí và dập dao động phụ tải trong cầu trục. Phương trình chuyển động bao gồm động học của dây tời và cơ cấu truyền động được tổng hợp thông qua nguyên lý Hamilton mở rộng. Luật điều khiển được thiết kế nhờ phương pháp Lyapunov trực tiếp và backstepping. Tính ổn định của hệ thống kín được chứng minh, một số mô phỏng cũng được đưa ra nhằm minh họa tính đúng đắn của nghiên cứu.

**Từ khóa**: Hệ mềm, hệ thống nâng hạ, điều khiển tựa từ thông, ổn đinh Lyalunov

## **1. INTRODUCTION**

Due to the flexibility in handling loads, gantry crane systems are essential in industrial and logistic applications. Expected operating condition of the gantry system is the desired positions of the trolley and the pay-load is coinciding. In practice, this is impossible because of swinging motion of the payload. Swinging payload phenomenon slows down goods handling operations and can be a potential threat to human and surrounding devices. Certain types of payload can ignite multi-modes or double-link pendulum effects [\[1\]](#page-4-0) [-\[4\].](#page-4-1) In addition, characterized as a class of under-actuated systems, precisely controlling trolley position and suppressing payload vibration simultaneously pose many challenges for control engineers.

In order to find a solution for the aforementioned control problem, a decoupling control law is proposed in [\[5\]](#page-4-2) to asymptotically stabilize trolley position and swing angle of the payload. Actually, the designed control only guarantee bounded swing angle. An improvement is made in [\[6\]](#page-4-3) with a gantry system with varying rope length. A switching control action is derived based on feedback lineariazation technique. Position control and vibration suppression of gantry crane is considered in [\[7\],](#page-4-4) the control problem is partly solved with the coupling effect between trolley and payload motions are taken into account. However, the obtained results are relatively limited in practice since the variance of system's parameters and actuator's dynamics are not considered. Practically, parameters of a gantry system is varying and challenging to identify due to hydrodynamic forces acting on the payload and the variance of payload mass and geometry. In order to deal with system uncertainty, an adaptive mechanism is integrated in proposed control law suggested in [\[8\].](#page-4-5) Well-known with its robustness against system uncertainty and disturbances, sliding mode control is applied in gantry control in [\[9\].](#page-4-6) However, it is need to cooperate with a pre-shape input to gain better performances [\[10\].](#page-4-7) Several adaptive schemes for gantry control also presented in [\[11\]](#page-4-8) and [\[12\].](#page-4-9)

Although having some promising results, feedback control seems to be suitable with automated gantry systems (where desired payload positions are predefined via a human machine interface). The reason is when in manual operating modes, human-in-the-loop actions may interfere with control decision of the feedback controller and deteriorate system responses. Instead of feedback controls, control actions from operators are modified before sending to the gantry actuators as shown in [\[13\]](#page-4-10) and [\[14\].](#page-4-11) The advantage of pre-shape input technique over feedback control is that measurement of system states is not required but a full knowledge of the system must be available. To rectify this drawback, pre-shape input method can be hybridised with a robust control as indicated in [\[14\]-](#page-4-11) [\[16\].](#page-4-12) A brief literature review above shows that the limitation of aforementioned researches rooted in the modeling step. Gantry control problem are solved with an assumption of pendulum motions of the payload that results in a system of ordinary differential equations govern system motions. Practice has shown that it is not the case, and gantry cable actually considered as a flexible systems whose

motions are modeled as a system of partial differential equations.

This paper directly designs a gantry control when taking flexibility of the cable into consideration. Trolley position and swinging payload suppression control is design based on Lyapunov's direct method. Finally stability of the closed-loop system is proven analytically and numerically.

#### **2. MATHEMATICAL MODEL**

An overhead crane system is illustrated in Fig. 1, where



**H. 1** *An overhead crane system.*

 $y(x,t)$  and  $y_d$  are the transverse motion of the crane's cable and target position of the payload, respectively.  $P_0$  is the cable tension,  $\rho$  is the mass per unit length of the cable, *L* is the length of the cable. *M* and *m* are trolley's and payload's masses, respectively. The force  $F(t)$  is generally generated by an induction motor. The kinetic energy of the system is

$$
T = \frac{M}{2} \left( \frac{\partial y}{\partial t} \right)^2 + \frac{\rho}{2} \int_0^L \left( \frac{\partial y}{\partial t} \right)^2 dx + \frac{m}{2} \left( \frac{\partial y}{\partial t} \right)^2, \quad (1)
$$

In (1) and from now onward the argument  $(x,t)$  is omitted for neat representation. In addition *y*(0) and  $y(L)$  are used to denote  $y(0,t)$  and  $y(L,t)$ , respectively. The potential energy can be expressed as

$$
P = \frac{P_0}{2} \int_0^L \left(\frac{\partial y}{\partial x}\right)^2 dx
$$
 (2)

where  $P_0$  is the tension of the cable. The work done by the external control force is given as

$$
W = F(t)y(L)
$$
 (3)

**Remark 1.** Bending stiffness of the cable is considerably small so that potential energy due to bending stiffness can be ignored. The cable is assumed to be inextensible. The extended Hamilton's principle is expressed as follows

$$
\int_{t_1}^{t_2} \delta T - P + W \ dx \tag{4}
$$

Substituting  $(1)$ ,  $(2)$  and  $(3)$  into  $(9)$  results

Substituting (1), (2) and (3) into (9) results  
\n
$$
\int_{t_1}^{t_2} \int_0^L \left[ -\rho \frac{\partial y^2}{\partial t'} + P_0 \frac{\partial^2 y}{\partial x^2} \right] \delta dx - P_0 \frac{\partial y}{\partial x} \Big|_0^L
$$
\n
$$
+ M \frac{\partial y^2(0)}{\partial t^2} \delta y(0) dt + m \frac{\partial y^2(0)}{\partial t^2} \delta y(0)
$$
\n
$$
+ F(t) \delta y(0) dt = 0
$$
\n(5)

Using integration by parts, the equations of motions and boundary conditions of the crane system can be given as

$$
\rho y_u - P_0 y_x = 0
$$
  
\n
$$
My_u(0) + P_0 y_x(0) = F(t)
$$
  
\n
$$
my_u(L) - P_0 y_x(L) = 0
$$
\n(6)

The force acting on the trolley  $F(t)$  can be calculated in term of the motor torque as

$$
F(t) = \frac{i\eta}{R_b} m_M , \qquad (7)
$$

where  $R_b$  is the radius of the drum, *i* is the transmission ratio of the gearbox and  $\eta$  is the efficiency of the transmission system. Mathematical model of the asynchronous motor can be written as follows

$$
\begin{split}\ni_{sd} &= -\left(\frac{1}{\sigma} + \frac{1-\sigma}{\sigma T_r}\right)i_{sd} + \omega_s i_{sq} + \frac{1-\sigma}{\sigma T_r} \psi_{rd}' \\
&+ \frac{1-\sigma}{\sigma} \omega \psi_{rd}' + \frac{1}{\sigma L_s} u_{sd}, \\
i_{sd} &= -\omega_s i_{sd} - \left(\frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r}\right)i_{sd} - \frac{1-\sigma}{\sigma} \omega \psi_{rd}' \\
&+ \frac{1-\sigma}{\sigma T_r} \psi_{rd}' + \frac{1}{\sigma L_s} u_{sq}, \\
\dot{\psi}_{rd}' &= \frac{1}{T_r} i_{sd} - \frac{1}{T_r} \psi_{rd}' + \omega_s - \omega \psi_{rd}', \\
\dot{\psi}_{rq}' &= \frac{1}{T_r} i_{sq} - \omega_s - \omega \psi_{rd}' - \frac{1}{T_r} \psi_{rq}', \\
m_M &= \frac{3}{2} K_m z_p \psi_{rd} i_{sq}.\n\end{split} \tag{8}
$$

In (8),  $i_{sd}$  and  $i_{sq}$  are direct and quadrature components of stator current.  $T_r$  and  $T_s$  are rotor and stator time constants,  $z_p$  is number of pole pairs, is total magnetic leakage factor.  $K_m = \frac{L_m}{L_r}$  $K_m = \frac{L_m}{L_r}$ , where  $L_m$ 

and  $L_r$  are mutual and rotor inductance.  $\psi'_{rd} = \frac{\psi_{ra}}{L_m}$ 

and 
$$
\psi'_{rq} = \frac{\psi_{rq}}{L_m}
$$
, where  $\psi_{rd}$  and  $\psi_{rq}$  are  $dq$ 

components of the rotor flux. The coupled electricalmechanical system is rewritten as follow

$$
\rho y_u - P_0 y_x = 0,
$$
  
\n
$$
My_u(0) - P_0 y_x(0) = \Omega i_{sq},
$$
  
\n
$$
\dot{i}_{sq} = -\theta_1 i_{sd} - \theta_2 i_{sq} - \theta_3 + \theta_4 u_{sq},
$$
  
\n
$$
my_u(L) + P_0 y_x(L) = 0.
$$
  
\nwhere  
\n
$$
\theta = \omega, \theta = \frac{1}{\omega} + \frac{1 - \sigma}{\omega} \theta = \frac{1 - \sigma}{\omega} \omega
$$

$$
\theta_1 = \omega_s, \theta_2 = \frac{1}{\sigma T_s} + \frac{1 - \sigma}{\sigma T_r}, \theta_3 = \frac{1 - \sigma}{\sigma} \omega \psi_{rd}'
$$

$$
\theta_4 = \frac{1}{\sigma L_s}, \Omega = \frac{3}{2} \frac{i\eta}{R_b} K_m z_p \psi_{rd}'
$$
(10)

**Remark 2.** Equation (9) is derived under a condition that the rotor flux orientation is obtained, i.e.,  $\psi'_{rq} = 0$ .

Moreover, it is assumed that  $i_{sd}$  and  $\psi'_{rd}$  are kept constants by current and flux controllers and their values are available for feedback. In addition, the current controller has the ability of decoupling  $i_{sd}$ and  $i_{sq}$ .

It is noted that from this point onward, argument *t* is ignored where applicable.

### **3. CONTROL DESIGN**

The control objective is to simultaneously stabilize the trolley and the payload at the desired position. An investigation of the system given in (9) shows that the system is of strict-feedback form. Hence, in this paper, backstepping technique will be employed to design the control input  $u_{sq}$ . The choice of backstepping as a design tools make it ready if system parameters adaptation is needed. The control design process comprises of two steps. In order to satisfy the control objective, at first we take  $i_{sq}$  as a control and define

$$
z = \Omega i_{sq} - \alpha \tag{11}
$$

where is  $\alpha$  virtual control. Consider the following

Lyapunov candidate function  
\n
$$
W = \frac{\rho}{2} \int_0^L y_t^2 dx + \frac{P_0}{2} \int_0^L y_x^2 dx + \frac{m}{2} y_t^2 (L) + \frac{M}{2} y_t^2 (0) + \frac{\Delta}{2} y(0) - y_d
$$
\n(12)

where  $\Delta$  is a strictly positive constant. It is straightforward to show that *V* can be lower and upper bounded as below<br> $W_{\lambda}$   $\left(\begin{array}{cc} \int_0^L & 2 \end{array} \right)$   $\left(\begin{array}{cc} \int_0^L & L \end{array} \right)$ 

$$
W \ge \gamma_1 \left( \int_0^L y_t^2 dx + \int_0^L y_x^2 dx + y_t^2 (L) + y_t^2 (0) + \left[ y \ 0 - y_d \right]^2 \right) \tag{13}
$$

and

$$
W \leq \gamma_2 \left( \int_0^L y_t^2 dx + \int_0^L y_x^2 dx + y_t^2 (L) + y_t^2 (0) + \left[ y \ 0 \ - y_d \right]^2 \right) \tag{14}
$$

where

$$
\gamma_1 = \frac{1}{2} \min \left\{ \int_0^L y_t^2 dx, \int_0^L y_x^2 dx, y_t^2(L), y_t^2(0), y(0) - y_d \right\}
$$
\n(15)

and

and  
\n
$$
\gamma_2 = \frac{1}{2} \min \left\{ \int_0^L y_t^2 dx, \int_0^L y_x^2 dx, y_t^2(L), y_t^2(0), y(0) - y_d \right\}
$$
\n(16)

Time derivative

Time derivative  
\n
$$
\dot{W} = P_0 y_t y_x \Big|_0^L - P_0 y_t (L) y_x (L) + y_t (0) \alpha_1 + z_1 + P_0 y_x (0) + \Delta [y \ 0 \ - y_d] y_t \ 0
$$
\n
$$
= y_t \ 0 \ \alpha + z + \Delta [y \ 0 \ - y_d] \tag{17}
$$

Equation (17) suggests that virtual control  $\alpha_1$  can be chosen as follows

$$
\alpha = -ky_t \quad 0 \quad -\Delta \left[ y \quad 0 \quad -y_d \right] \tag{18}
$$

where k is a strictly positive constant. In the second step, the actual control input  $u_{sq}$  is designed to regulate  $z_1$  at the origin. To archive this target, we consider a Lyapunov candidate function

$$
V = W + \frac{1}{2}z^2\tag{19}
$$

Time derivative

Time derivative  
\n
$$
\dot{V} = -k_1 y_t^2 \quad 0 + z \left[ -\frac{\theta 1}{\Omega 1} i_{sd} - \frac{\theta 2}{\Omega} i_{sq} - \frac{\theta 3}{\Omega} + \frac{\theta 4}{\Omega} u_{sq} \right]
$$
\n
$$
-k_1 y_u \quad 0 - \Delta - 1 \quad y_t \quad 0 \quad \bigg]
$$

The actual control input  $u_{sq}$  can be derived as

$$
\frac{\theta_4}{\Omega} u_{sq} = \frac{\theta_1}{\Omega_1} i_{sd} + \frac{\theta_2}{\Omega} i_{sq} + \frac{\theta_3}{\Omega} + k_1 y_n \quad 0
$$
\n
$$
+ \Delta - 1 y_t \quad 0
$$
\n(21)

where  $k_2$  is positive constant.

**Remark 3:** The control input  $u_{sd}$  can be derived

based on backstepping control method as follows  
\n
$$
\frac{1}{s}u_{sd} = \frac{1}{T\sigma}i_{sd} - \omega_s i_{sq} - \frac{1-\sigma}{r}\hat{\psi}_{rd}'
$$
\n
$$
+ \left(\frac{1}{T_r} - c_1\right)i_{sd} - \hat{\psi}_{rd}' + c_1T_r\frac{d\hat{\psi}_{rd}'}{dt}
$$
\n
$$
+ T_r\frac{d^2\hat{\psi}_{rd}'}{dt^2} - c_2z_2 - \frac{1}{T_r}z_1 - d_2z_2\theta.
$$
\n(22)

where

$$
\theta_2 = \left(\frac{1-\sigma}{\sigma T_r}\right)^2 + \left(\frac{1-\sigma}{\sigma}\omega\right)^2\tag{23}
$$

 $c_1$  and  $c_2$  are strictly positive constant,  $z_1$  and  $z_2$  are errors between desired and virtual control when designing the flux controller.

The control design is complete and it is straightforward to to show that with the selected control input  $u_{sq}$  render the first time derivative of the

Lyapunov candidate function 
$$
\vec{V}
$$
 as

$$
\dot{V} = -k_1 y_t(0) - k_2 z \le 0 \tag{24}
$$

Inequality () shows that  $V(t)$  is upper bounded by *V*(0). This consequently implies that  $y_t(L)$ ,  $y_t(0)$ and  $y_t(0) - y_d$  are bounded. Further investigation of () can prove exponential convergence to  $y_d$  of  $y(0)$ and  $y(L)$ .

### **4. NUMERICAL SIMULATIONS**

To verify the effectiveness of the proposed control. Simulations are carried out using an asynchronous motor of 7.5kW (other motor power ranges can be applied with no loss of generality). The closed loop system is simulated in Matlab/Simulink environment. Simulation parameters are given in Table 1.

**Table 1.** *Induction motor parameters*

<b>Parameter</b>	Value
Nominal power	$PN = 7,5kW$
Pairs of pole	$Pc = 2$
Nominal current	$UN = 340V$
Nominal speed	$n = 1400$ rpm
Stator resistance	$RS = 2.521\Omega$
Stator inductance	$LS = 0.1825$ H
Rotor resistance	$RS = 0.976$
Rotor inductance	$LR = 0.1858 H$
Inertial moment	$J = 0,117$ kg.m2

The mass of the trolley and the payload are 500kg and 1000kg, respectively. The cable length is of 10m. Simulation scenario is to regulate trolley and payload position at a position of 10m from the initial condition. It is assumed that initially the positions of the trolley and payload are coinciding.



**H. 2** *System response of the trolley, the flexible cable and the payload.*









Numerical sumulation indicates that the effectiveness of the proposed control design. Trolley position is regulated at the desired value after 6s, and the payload also reach the target after a few oscilations. In addition, the control input is of the applicatble in practice range.

#### **5. CONCLUSION**

A design of a position and vibration suppression control of a gantry crane system is designed in this paper. Based on energy approach, a system of partial differential and ordinary differential equations that govern the system's motions including cable and actuator dynamics are derived. The Lyapunov direct method and backstepping technique are employed to design the controller. Stability and the effectiveness of the closed-loop system are verified analytically and illustrated numerically.

### **Acknowledgment**

*The authors would like to thank Institute for Control Engineering and Automation (ICEA), Hanoi University of Science and Technology for supporting this work.*

## **References**

- <span id="page-4-0"></span>[1] [1] R. Blevins, *Formulas for Natural Frequency and Mode Shape*. New York, NY: Van Nostrand Reinhold Co., 1979.
- [2] W. Singhose, D. Kim, and M. Kenison, *"Input shaping control of double-pendulum bridge crane oscillations,"* Journal of Dynamic Systems, Measurement, and Control, vol. 130, no. 3, pp.  $1 - 7$ , May 2008.
- [3] R. Manning, J. Clement, D. Kim, and W. Singhose, *"Dynamics and control of bridge cranes transporting distributed-mass payloads,"* J. Dyn. Syst. Meas. Control (USA), vol. 132, no. 1, pp. 014 505 (8 pp.), 2010/01/.
- <span id="page-4-1"></span>[4] D. Kim and W. Singhose, *"Performance studies of human operators driving double-pendulum bridge cranes,"* in Control Engineering Practice, vol. 18, no. 6, pp. 567 – 576, 2010.
- <span id="page-4-2"></span>[5] Y. Fang, W. Dixon, D. Dawson, and E. Zergeroglu, *"Nonlinear coupling control laws for an under-actuated overhead crane system,"* in IEEE/ASME Trans. Mechatronics, vol. 8, no. 3, pp. 418–423, Sep. 2003.
- <span id="page-4-3"></span>[6] H. Park, D. Chwa, and K. S. Hong, "A feedback linearization control of container cranes: Varying rope length," Int. J. Control Autom. Syst., vol. 5, no. 4, pp. 379–387, Aug. 2007.
- <span id="page-4-4"></span>[7] H. H. Lee, *"An anti-swing trajectory control of overhead cranes with high-speed hoisting,"* in Proc. Amer. Control Conf., Denver, CO, Jun. 2003, pp. 1440–1445.
- <span id="page-4-5"></span>[8] H. Lee, Y. Liang, and D. Segura, *"A new approach for the anti-swing control of overhead cranes with high-speed load hoisting,"* in Int. J. Control, vol. 76, no. 15, pp. 1493–1499, Oct. 2003.
- <span id="page-4-6"></span>[9] H. Lee, Y. Liang, and D. Segura, *"A slidingmode anti-swing trajectory control for overhead cranes with high-speed load hoisting,"* Trans. ASME, J. Dyn. Syst. Meas. Control, vol. 128, no. 4, pp. 842–845, Dec. 2006.
- <span id="page-4-7"></span>[10] H. Lee, *"Motion planning for three-dimensional overhead cranes with high-speed load hoisting,"*  Int. J. Control, vol. 78, no. 12, pp. 875–886, Aug. 2005.
- <span id="page-4-8"></span>[11] J. H. Yang and K. S. Yang, *"Adaptive coupling control for overhead crane systems,"* in Mechatronics, vol. 17, no. 2/3, pp. 143–152, Mar./Apr. 2007.
- <span id="page-4-9"></span>[12] J. H. Yang and K. S. Yang, *"Adaptive control for 3-D overhead crane systems,"* in Proc. Amer. Control Conf., Minneapolis, MN, Jun. 2006, pp. 1832–1837.
- <span id="page-4-10"></span>[13] O. Smith, *Feedback Control Systems*, New York : McGraw-Hill Book Co., Inc., 1958.
- <span id="page-4-11"></span>[14] N. C. Singer and W. P. Seering, *"Preshaping command inputs to reduce system vibration,"* J. of Dynamic Systems, Measurement, and Control, vol. 112, pp. 76–82, March 1990.
- [15] W. Singhose, W. Seering, and N. Singer, *"Residual vibration reduction using vector diagrams to generate shaped inputs,"* ASME J. of Mechanical Design, vol. 116, pp. 654–659, June 1994.
- <span id="page-4-12"></span>[16] T. Singh and S. R. Vadali, *"Robust time-delay control,"* J. of Dynamic Systems, Measurement, and Control, vol. 115, pp. 303–6, 1963.
- [17] A. Khalid, J. Huey, W. Singhose, J.Lawrence, and D.Frakes,*"Human operator performance testing using an input-shaped bridge crane,"* J. of Dynamic Systems, Measurement and Control, vol. 128, no. 4, pp. 835 – 841, 2006.
- [18] D. Kim and W. Singhose, *"Manipulation with tower cranes exhibiting double-pendulum oscillations,"* in IEEE International Conference on Robotics and Automation, Rome, Italy, 10- 14 April 2007.
- [19] R. Manning, D. Kim, and W. Singhose, *"Reduction of distributed payload bridge crane oscillations,"* in WSEAS Int. Conf. on Automatic Control, Modeling, and Simulation, Instanbul, Turkey, 2008



**Tung Lam Nguyen** received his B.S. degree in Electrical Engineering from Hanoi University of Science and Technology, Vietnam, in 2005. He received his M.S. degree from Asian Institute of Technology, Thailand and Ph.D from The

University of Western Australia in 2007 and 2014, respectively. He is current is with Department of Industrial Automation, Hanoi University of Science and Technology. His research interests are in the areas of nonlinear control, flexible systems.



**Duong Minh Duc** was born in 1979. He graduated from Department of Industrial Automation, Hanoi University of Technology in 2002. From 11- 2002, he becomes lecturer at Hanoi University of Technology. He received Master degree in Production System Eng. and Doctor degree in Electronic and

Information Eng at Toyohashi University of Technology, Japan in 2005 and 2008, respectively. His current research fields include bilateral teleoperation, vibration suppression control and rehabilitation robot system.