Kinematic Analysis for Working Space of the Rotopod Mechanism

Phân tích động học không gian làm việc của cơ cấu Ropotod

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Abstract

The mechanisms of parallel structure mechanisms are widely deployed in different areas. The industry of training simulators is one of the areas where the usage of such mechanisms is most common and effective solution. This paper presents the conceptual of a rotational parallel mechanism (Rotopod mechanism). This mechanism is used to reproduce movement, inverse kinematic, the working space were analysed.

Computational tools were used to resolve equations, create a virtual design in the conceptual design. The rotational parallel robot used six servomotors and six links with spherical joints that provide 360° rotational capabilities on its axis. That mean, the working space of the rotopod mechanism - the universal base for a 6-DoF training simulator. The purpose is to achieve an increase in functionalities of parallel mechanisms in view of their possible singularities.

Keywords: Rotopod mechanism, parallel mechanism, kinematic analysis, working space, singularity.

Tóm tắt

Các cơ cấu có cấu trúc song song được triển khai rộng rãi trong các lĩnh vực khác nhau. Ngành công nghiệp mô phỏng huấn luyện là một trong những lĩnh vực mà việc sử dụng các cơ cấu như vậy là giải pháp phổ biến nhất và có hiệu quả. Bài báo trình bày các ý tưởng của một cơ cấu song song quay (cơ cấu Rotopod). Cơ cấu song song được sử dụng để tái tạo chuyển động, động học ngược, không gian làm việc đã được phân tích.

Công cụ tính toán được sử dụng để giải các phương trình, tạo ra một thiết kế ảo trong ý tưởng thiết kế. Robot song song quay sử dụng sáu mô tơ và sáu liên kết với các khớp cầu để cung cấp khả năng quay 360° quanh trục của nó. Nghĩa là, không gian làm việc của các cơ cấu Rotopod – có 6 bậc tự do chuyển động của mô phỏng. Mục đích là để đạt được sự gia tăng chức năng của các cơ cấu song song khi tính đến tính kỳ dị của cơ cấu.

Từ khóa: Cơ cấu Rotopod, cơ cấu song song, phân tích động học, không gian làm việc, tính kỳ dị.

1. Introduction

In recent years, numerous researchers have investigated the parallel mechanisms and many studies

have been done on the kinematics or dynamics analysis. Here, let us mention only some books in which parallel mechanisms are considered by [2-3, 12]. Reference [4] has given the singularity criteria based on Jacobian matrices when describing the various types of singularity. Then, in [10] proposed other singularity criterion for consideration of these problems the screw theory based on the approach of the [5]. This criterion is determined by the constraints imposed by the kinematic chains, as in [1, 10]. Taking into account some problems the Plücker coordinates of constraint wrenches can be applied by [14-16, 18].

Singularities configuration of the parallel mechanisms have been discussed in [4], [6-9], [12-13, 19] that considered to take into account constraints restricting working space. It is necessary to develop the means of design automation of such mechanisms in which the optimal structure and parametrical properties have combined [17].

The contribution of this paper is to express the constraints existing in known parallel mechanisms in another form, as a parallel rotational mechanism is rotopod mechanism by kinematics analysis. The rotopod is a novel robot mechanism that combines the features of wheeled and legged locomotion in an unusual way. This robot has the advantage of legged locomotion in stepping its *1-DOF* legs over objects, but its drive mechanism is a rotating reaction mass that rotates the robot, in a controllable fashion, around each of its legs, similar to a rotating wheel. The mechanism has the potential to transfer the energy from the rotating reaction mass in an efficient manner to the legs, effecting a spinning forward motion.

2. Rotopod structural analysis

The Rotopod mechanism is a *6-DoF* mechanism with six kinematic chains. Each kinematic chain consists of a link with two ball joints on the ends of it and a joint with a drive, all drive joints axis are in one place - so in practice they are implemented as carriages (see in Fig. 1).



Fig. 1. The Rotopod mechanism

Lets start with defining the notation: A_1 , A_2 , $A_3 A_4$, A_5 , A_6 are the ball joints connected to the output link B_1 , B_2 , B_3 , B_4 , B_5 , B_6 - are the carriages, L is the length of the kinematic chain link, R is the radius of the base, r is the radius of the output link platform. To have a simple illustration we shall define their values: L = 0.98 (*m*); R = 1.0198 (*m*); r = 0.95 (*m*).

The first task to find out the working space of the mechanism is to find angle carriage positions φ_1 , φ_2 , φ_2 , φ_4 , φ_5 , φ_6 that comply the output link positions. For the beginning, we must find the matrix to transition the co-ordinates of the output link to co-ordinates of the base. We assume this transition as a sequence of movements: rotation α around *OX* axis, rotation β around *OY* axis, rotation γ around *OZ* axis and then movement x_1 , y_1 , z_1 along *OX*, *OY*, *OZ* accordingly.

The matrix defining rotation α around *OX* axis:

1	1	0	0	0\						
l	0	$\cos(\alpha)$	$-\sin(\alpha)$	0						
l	0	sin(α)	$\cos(\alpha)$	0						
	0	0	0	1/						
e matrix defining rotation β around OY axis:										
1	΄ cα	$os(\beta)$	0 sin((β) 0 \						

0	1	0	0	
$-\sin(\beta)$	0	cos(β)	0	
0	0	0	1	/

The matrix defining rotation γ around *OZ* axis:

$(\cos(\gamma))$	$-\sin(\gamma)$	0	0		
$sin(\gamma)$	$\cos(\gamma)$	0	0		
0	0	1	0		
0	0	0	1	/	

The matrix defining movement x_1 , y_1 , z_1 along *OX*, *OY*, *OZ*:

 $\begin{pmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Therefore, the transitional matrix we are looking for is the multiplication of these matrixes:

$$M = \begin{pmatrix} \cos(\gamma) \cdot \cos(\beta) & \cos(\gamma) \cdot \sin(\beta) \cdot \sin(\alpha) - \sin(\gamma) \cdot \cos(\alpha) \\ \sin(\gamma) \cdot \cos(\beta) & \cos(\gamma) \cdot \cos(\alpha) + \sin(\gamma) \cdot \sin(\beta) \cdot \sin(\alpha) \\ -\sin(\beta) & \cos(\beta) \cdot \sin(\alpha) \\ 0 & 0 \\ \sin(\gamma) \cdot \sin(\alpha) + \cos(\gamma) \cdot \cos(\alpha) \cdot \sin(\beta) & x_1 \\ \sin(\gamma) \cdot \cos(\alpha) \cdot \sin(\beta) - \cos(\gamma) \cdot \sin(\alpha) & y_1 \\ \cos(\beta) \cdot \cos(\alpha) & z_1 \\ 0 & 1 \end{pmatrix}$$

The matrix defining the coordinates of ball joints centers A_i (*i*=1,2,...,6) in the output link co-ordinates:



So these coordinates in the base co-ordinates can be found as:

$$A = M \cdot P_1$$

Lets define the φ_1 , φ_2 , φ_3 , φ_4 , φ_5 , φ_6 angles as angles between the OX axis and radius-vectors OB_i (i=1,2,...,6) to B_1 , B_2 , B_3 , B_4 , B_5 , B_6 .







Fig. 3 The base geometry

The angle φ_{11} (Fig. 3), is the angle between radius vector l_{12} (A_1 projection to XOY) and the OX vector. We must consider the A_1 position on the XOY finding this angle if $Y_{A1} > 0$, then $\varphi_{11} = \arccos\left(\frac{x_{A1}}{l_{12}}\right)$,

The

if $Y_{A1} < 0$, then $\varphi_{11} = 2\pi - \arccos\left(\frac{X_{A1}}{l_{12}}\right)$, l_{12} is the distance between the *XOY* co-ordinates center and A_1 projection to *XOY* (Fig. 3):

 $l_{12} = \sqrt{X_{A1}^2 + Y_{A1}^2}.$ So from the triangle *OAB* (Fig. 3) angle φ_{12} : $\varphi_{12} = \arccos\left(\frac{R^2 + l_{12}^2 - l_{11}^2}{2R \cdot l_{12}}\right),$ $l_{11} = \sqrt{L^2 - Z_{A1}^2}$ (in Fig. 4). $\varphi_i = \varphi_{i1} \pm \varphi_{i2}$

Fig. 4. Co-ordinate system and mechanism geometry From this equation, we have two possible positions for B_i - it depends on how we count φ_{12} clockwise B_1 or counterclockwise B'_1 (see in Fig. 3). In the deployment the mechanism can't have the positions where its links are crossed and carriages can not overlap - therefore we assume B_i with odd indexes have φ_{12} clockwise and with even counterclockwise as a bonus we get symmetrical positioning of the chains and stability of the rotopod.

3. The limiting criteria while finding working space

We found the solution that allows finding the working space of theoretical rotopod model but in real life applications there are some additional limitations.

- The links length.
- The maximum and minimum distance between the carriages on the base.
- The impossibility of carriages crossing or overlapping.
- The maximum and minimum angles between the links and the base.
- The maximum and minimum angles between the links and the output link.
- Singularities check.
- The impossibility of output link mass center position outside the base carriages circle.

So let us find the limiting criteria. The first check is simple the output link joints A_i (*i*=1,2,...,6) can not be higher than their length so the criteria is $Z_{Ai} < L$.

Analyzing maximum and minimum distance between the carriages on the base, we shall consider this distance as an angle. Therefore, for the minimum distance we shall look through the vector multiplication of OB_1 and OB_2 , vectors

$$V12 = \begin{pmatrix} 0 & 0 & \sin(\varphi_1) \\ 0 & 0 & -\cos(\varphi_1) \\ -\sin(\varphi_1) & \cos(\varphi_1) & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos(\varphi_2) \\ \sin(\varphi_2) \\ 0 \end{pmatrix}$$

 $UV12 = \arcsin(V12_3)$, UV12 is the angle between B_1 and B_2 in radians for our example we shall define the minimum angle to be 12 degrees. Therefore, we have the criteria:

$$UV12 = \frac{180^{\circ}}{\pi} \operatorname{arcsin}(V12_3) \le 12^{\circ}.$$

In addition, this check can be used to track the carriages overlapping and crossing - V12 must be more than zero.

For the maximum distance we shall consider scalar multiplication of these vectors (we must note that the result changes its from positive to negative passing 90° .

$$S12 = \begin{pmatrix} \cos(\varphi_1) \\ \sin(\varphi_1) \\ 0 \end{pmatrix}^T \cdot \begin{pmatrix} \cos(\varphi_2) \\ \sin(\varphi_2) \\ 0 \end{pmatrix},$$

For our example, we shall define the maximum angle to be *120* degrees. Therefore, we have the criteria:

$$US12 = \frac{180}{\pi} \arccos(S12) \le 120^\circ.$$

The minimum angle between the links $(A_iB_i \text{ vector})$ and the base we shall find through the scalar multiplication of OZ unit vector and A_iB_i vector

The matrix with B coordinates in the base coordinates system:

$$B = \begin{pmatrix} R\cos(\varphi_1) & R\cos(\varphi_2) & R\cos(\varphi_3) \\ R\sin(\varphi_1) & R\sin(\varphi_2) & R\sin(\varphi_3) \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ R\cos(\varphi_4) & R\cos(\varphi_5) & R\cos(\varphi_6) \\ R\sin(\varphi_4) & R\sin(\varphi_5) & R\sin(\varphi_6) \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ K = A - B. \end{cases}$$

K columns are formed by $A_i B_i$ vectors (we must note that the 4-th row is auxiliary) for our example, we shall define the maximum angle to be 11 degrees so we have the criteria:

$$ug11 = \left(\frac{\pi}{2} - \arccos\left(\frac{1}{L} \begin{pmatrix} K_{11} \\ K_{21} \\ K_{31} \end{pmatrix}^T \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)\right) \frac{180^*}{\pi} \le 11^\circ \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

is the OZ unit vector.

The maximum and minimum angles between the links $(A_iB_i \text{ vector})$ and the output link we shall find through the scalar multiplication of OZ' unit vector and A_iB_i vector the same way as with the base for our example we shall define the maximum angle to be 22 degrees so we have the criteria:

$$ug12 = \left(\frac{\pi}{2} - \arccos\left(\frac{1}{L}\binom{K_{11}}{K_{21}}^T \cdot \binom{Z'_x}{Z'_y}\right)\right) \frac{180^\circ}{\pi} \le 22^\circ$$

Vector
$$\begin{pmatrix} Z'_x \\ Z'_y \\ Z'_z \end{pmatrix}$$
 is the unit vector he *OZ'* axis
 $OZ' = M \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$, M is the transitional matrix

found above.

4. Working space while taking into account singularities

Next, Influence of singularities on parameters of the working space of the parallel mechanism is a significant factor worth investigating. In these singularity configurations, lost control, it acquires more mobility and greatly affects the functionality of the devices. It is necessary to analyze how the criteria value of existence of the singularity configuration affects the parameters of the working space that is necessary to find the limits of the possible values of those criteria.

We need to verify the proximity to the position singular of the matrix made from Plücker coordinates of the unit vectors directed along the axes of the six legs for each position.

Let us consider a parallel mechanism (Fig. 1.). Singularities of the mechanism is determined by closeness to zero of determinant det(E) of the matrix (E), the matrix made from Plücker coordinates unit screws E_i , (i=1, ..., 6) of axes of wrenches acting from kinematic chains to the output link. These coordinates form the (6×6) matrix (E):

$$E = \begin{pmatrix} X_1 & Y_1 & Z_1 & X_1^0 & Y_1^0 & Z_1^0 \\ X_2 & Y_2 & Z_2 & X_2^0 & Y_2^0 & Z_2^0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ X_6 & Y_6 & Z_6 & X_6^0 & Y_6^0 & Z_6^0 \end{pmatrix},$$

All the matrix elements are the Plücker coordinates of the unit vectors the unit screw, going through link A_1B_1 (first row), we can find the coordinates:

$$(X_1, Y_1, Z_1) = (A_1 B_{1x}/L, A_1 B_{1y}/L, A_1 B_{1z}/L)$$

$$A_1B_{1x'}$$
, $A_1B_{1y'}$, A_1B_{1z} are defined by the K matrix.

$$(X_1^0, Y_1^0, Z_1^0) = \begin{pmatrix} 0 & -B_{21} & B_{21} \\ B_{31} & 0 & -B_{11} \\ -B_{21} & B_{11} & 0 \end{pmatrix} \cdot \begin{pmatrix} A_1 B_{1x} \\ A_1 B_{1y} \\ A_1 B_{1z} \end{pmatrix}$$

for other links, we can calculate it the same way the unit vectors Plücker coordinate matrix determinant closeness to the zero is our criteria for singularities det(E) > 0.

In addition, the last check - the mass center projection must be within the hexahedron formed by B_1 , B_2 , B_3 , B_4 , B_5 , B_6 . The gain of this check is to prevent appearance of the overturn moment caused by platform weight.

The question arises, how to choose the singular criterion. In this section, we is used the criterion that

operates with the coordinates axes linear motors of the mechanism. Thus, it is necessary to consider the matrix composed of these coordinates. Coordinates axes linear motors with one to four are defined as follows. We must take a unit vector, which is located along the corresponding axis, and find three of its coordinates as normal vector. Those three coordinates define the direction of the vector. The other three coordinates needed to establish the position of the axis in space; we find a vector product of the radius vector of any point on the axis of the unit vector of the axis. These moments define the vector product of unit vector axis relative to the origin.

Here, vector G is the center mass vector in X'Y'Z' co-ordinate system.

$$G = \begin{pmatrix} G_x \\ G_y \\ G_z \\ 1 \end{pmatrix}.$$

going to the XYZ co-ordinate system. $G_o = M \cdot G$.

Then we calculate the vector multiplication of B_1B_2 vector (chord between B_1 and B_2) by B_1C vector, *C* is the mass center projection to *XOY*.

Therefore, if the multiplication is more than zero *C* is located to the left from B_1B_2 and our criteria are:

 $B_1B_2 x B_1C > 0.$

We must check all the chords the same way.

Using this data we can scan all the amount of the output link 6-DoF positions in the potential working space and using the criteria negate the impossible positions getting the real life rotopod working space.

The result of such testing can be seen on Fig. 5 with example parameters we mentioned above (L = 0.98 (m); R = 1.0198 (m); r = 0.95 (m)) - the scanning was made with parameters is OX axis step 0.05 (m), OY axis step 0.1 (m), OZ axis step 0.01 (m) and no rotation about axis.

In Fig. 5., (A), (B) and (C) section show the projection of the scan to the *ZOY*, *ZOX*, *XOY* the (D) section shows the overall view.

When the value of the criterion that determines the proximity to singular configurations is equal to zero, we can assume that the constraints associated with the singularity in general, are not imposed in the analysis of each specific configuration.

Limiting possible module of a determinant of the matrix (E) to singularity configurations changes the Plücker coordinates of the wrenches transmitted on the output link. Methodology for analyzing the singularities on optimization appearing in the parallel mechanism and their impact in the working space is proposed. The practical significance from the fact is the results obtained in this work increase the effectiveness of design automation.



Fig. 5. The result of such testing

5. Conclusions

In this paper, the constraints existing in known parallel mechanism in another form as a parallel rotational mechanism is rotopod mechanism. It allows specifying the results of structural analysis and parametrical synthesis of the parallel mechanisms.

This mechanism is used to reproduce movement, inverse kinematic, the working space was considered. By mean of kinematic analysis of the output link, we obtained the graphs of position and angle needed to assess possible behavior of the actuator in defined path then determine the working space of the rotopod mechanism. Working space of the rotopod mechanism is limited according to the relation between the ratio of the diameter of circular guide and the length of the links. The paper also proposes a methodology for analyzing the singularities appearing in the parallel mechanism and their impact in the working space.

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