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COMPOSITIONAL RULE OF CHAIN INFERENCE IN COMPUTATIONAL INTELLIGENCE PROBLEMS

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ABSTRACT: *The incorporation of imprecise, linguistic information into logical deduction processes is a significant issue in computational intelligence. Throughout the literature, we can find all sorts of intelligent inference schemes acting under imprecision; common to most approaches is their reliance on if-then rules of the kind "IF X is A THEN Y is B", where A and B are fuzzy sets (FS) in given universes U and V. While the FS-based theory of approximate reasoning is surely a well-established and commonly applied one, there is still for further expanding the expressiveness of the formalism. One such improvement can be obtained by using picture fuzzy sets (PFS), in which the sets A and B are picture fuzzy sets in the corresponding universes U and V. In this paper, we will contribute to the further development of the picture fuzzy logic (PFL) by presenting some new classes of implication operators in PFL and firstly defining the Compositional Rule of Chain Inference (CRCI) in a PFL setting. The new chain inference procedures should be applied in computational intelligence problems*

Keywords*: Picture fuzzy set, composition rule of chain inference, inference procedure, implication operator*.

I. INTRODUCTION

Inference is defined as a procedure for deducing new facts out of existing ones on the basis of formal deduction rules. Classical paradigms like two-valued propositional and predicate logic, exhibit some important drawbacks that make them unsuitable for application in automated deduction systems (e.g. for medical diagnosis). To alleviate these difficulties, Zadeh in 1973 [6] introduced a formalism called approximate reasoning to cope with problems which are too complex for exact solution but which do not require a high degree of precision.

From a logical perspective, it is interesting to see how people are able to combine such information efficiently in a Modus Ponens- like fashion to allow for inferences of the following kind:

IF bath water is "too hot"	THEN	Mrs Wang is apt to get burnt
Bath water is "really rather hot"		
		Mrs Wang is quite apt to get burnt

With his introduction of a calculus of fuzzy operators Zadeh paved the way towards a reasoning schema called Generalized Modus Ponens (GMP) to systematize deductions like the example we presented. Since his pioneering work, many researchers have sought for efficient realizations of this approximate inference scheme. In this paper, we will contribute to the further development of the picture fuzzy logic (PFL) by presenting some new classes of implication operators in PFL and generalizing the well-known Compositional Rule of Inference (CRI) by the first defined Compositional Rule of Chain of Inference in a PFL setting. In Section 2 we will recall the notion of Picture Fuzzy Sets (PFSs) and some their connectives in Picture Fuzzy Logic (PFL). Section 3 is devoted some new classes of implication operators in PFL. In section 4 we will formally define the Composional Rule of Chain Inerence (CRCI) and some concrete computing of the new inference procedures. In fact, we proceed to extend the CRI to a PFS setting and firstly controduced the new compositional rule of chain inference. Finally, section 5 offers some options for future research.

II. PICTURE FUZZY SETS

In 2013, we introduced a new notion of picture fuzzy sets (PFS), which are direct extensions of the fuzzy sets [3, 4] and the intuitionistic fuzzy sets [1, 2]. Then some operations on PFS with some properties are considered in [7,8]. Some basic connectives of the picture fuzzy logic (PFL) as negation, t-norms, t-conorms for picture fuzzy sets are defined and firstly studied in [9, 10].

A. Some basic definitions of the picture fuzzy sets

Definition 2.1 [7] *A picture fuzzy set* A *on a universe* X *is an object of the form*

$$A = \left\{ (x, \mu_A(x), \eta_A(x), \nu_A(x)) \middle| x \in X \right\},$$

where $\mu_A(x) \in [0,1]$ *is called the "degree of positive membership of x in A",* $\eta_A(x) \in [0,1]$ *is called the "degree of neutral membership of x in A" and* $\nu_A(x) \in [0,1]$ *is called the "degree of negative membership of x in A", and where* μ_A*,* η_A *and* ν_A *satisfy the following condition:*

$$(\forall x \in X) \quad (\mu_A(x) + \eta_A(x) + \nu_A(x) \le 1),$$

and $1-(\mu_A(x)+\eta_A(x)+\nu_A(x))$ *could be called the "degree of refusal membership" of x in A.*

Let $PFS(X)$ denote the set of all the picture fuzzy sets on a universe X.

Basically, picture fuzzy sets based models may be adequate in situations when we face human opinions involving more answers of the type: yes, abstain, no, refusal.

Voting can be a good example of such a situation as the human voters may be divided into four groups of those who: vote for, abstain, vote against, refusal of the voting.

PFS is a direct generalization of the fuzzy set was introduced by Zadeh 1965 [5] and the intuitionistic fuzzy set was proposed by Atanassov 1983 [1].

Definition 2.2 [1] *A intuitionistic fuzzy set* A *on a universe* X *is an object of the form*

$$A = \{(x, \mu_A(x), \nu_A(x) | x \in X)\},$$

where $\mu_A(x) \in [0,1]$ *is called the "degree of membership of x in A",* $\nu_A(u) \in [0,1]$ *is called the "degree of non-membership of x in A"* **,** *and* μ_A *and* ν_A *satisfy the following condition*: $(\forall x \in X) \quad (\mu_A(x) + \nu_A(x) \le 1)$.

Let X, Y and Z be ordinary non-empty sets. An extension for picture fuzzy relations is the following:

Definition 2.3 [8] *A picture fuzzy relation is a picture fuzzy subset of* $X \times Y$*, i.e.* R *given by*

$$R = \{((x,y), \mu_R(x,y), \eta_R(x,y), \nu_R(x,y)) | x \in X, y \in Y)\}$$

where $\mu_R : X \times Y \to [0,1]$, $\eta_R : X \times Y \to [0,1]$, $\nu_R : X \times Y \to [0,1]$ *satisfy the condition*

$$0 \le \mu_R(x,y) + \eta_R(x,y) + \nu_R(x,y) \le 1 \text{ for every } (x,y) \in (X \times Y).$$

The set of all the picture fuzzy subsets in $X \times Y$ *is denoted by* $PFR(X \times Y)$**.**

The composition $P \circ E \in PFR(X \times Z)$ *of picture fuzzy relations* $E \in PFR(X \times Y)$ *and* $P \in PFR(Y \times Z)$ *was given in [8].*

B. Some Picture Logic Operators for PFSs

Consider the set $D^* = \{x = (x_1, x_2, x_3) \mid x \in [0,1]^3, x_1 + x_2 + x_3 \le 1\}$. From now on, we will assume that if $x \in D^*$, then x_1, x_2 and x_3 denote, respectively, the first, the second and the third component of x, i.e., $x = (x_1, x_2, x_3)$. We have a complete lattice $(D^*, \le_1)$ defined by

$$\forall x, y \in D^*:$$
$$x \le_1 y \Leftrightarrow \{x_1 < y_1, x_3 \ge y_3\} \cup \{x_1 = y_1, x_3 > y_3\} \cup \{x_1 = y_1, x_3 = y_3, x_2 \le y_2\},$$
$$x = y \Leftrightarrow \{x_1 = y_1, x_3 = y_3, x_2 = y_2\}.$$

Let $x, y \in D^*$, $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$. Denote $I(x) = \{y = (x_1, y_2, x_3) : 0 \le y_2 \le x_2\}, \forall x, y \in D^*$.

Picture fuzzy negation operators form an extension of fuzzy negations [3] and intuitionistic fuzzy negation operators [2], and are defined as follows.

Definition 2.4 *A picture fuzzy negation operator is a non-increasing mapping* $N : D^* \to D^*$*, satisfying* $N(0_{D^*}) = 1_{D^*}$*, and* $N(1_{D^*}) = 0_{D^*}$*, if* $N(N(x)) = x$*, for all* $x \in D^*$*, then* N *is called an involutive negation operator*.

The mapping $N_0 : D^* \to D^*$, defined by $N_0(x) = (x_3, 0, x_1)$, for all $x \in D^*$, is a picture negation operator. From now on, if $x = (x_1, x_2, x_3) \in D^*$, then $x_4 = 1 - (x_1 + x_2 + x_3)$, the mapping N_S defined by $N_S(x) = (x_3, x_4, x_1)$, for all $x \in D^*$, will be called the standard negation operator.

Definition 2.5 [9] *A mapping* $T: D^* \times D^* \to D^*$ *is a picture fuzzy t-norm if* T *satisfies the following conditions:*

1. $T(x,y) = T(y,x), \ \forall x, y \in D^*.$
2. $T(x, T(y,z)) = T(T(x,y), z), \forall x, y, z \in D^*.$
3. $T(x,y) \leq_1 T(x,z), \forall x, y, z \in D^*, y \leq_1 z.$
4. $T(x, 1_{D^*}) \in I(x), \ \forall x \in D^*.$

Definition 2.6 [9] *A mapping* $S: D^* \times D^* \to D^*$ *is a picture fuzzy t-conorm if* S *satisfies all following conditions:*

1. $S(x,y) = S(y,x), \forall x, y \in D^*.$
2. $S(x, S(y,z)) = S(S(x,y), z), \forall x, y, z \in D^*.$
3. $S(x,y) \leq_1 S(x,z), \forall x, y, z \in D^*, y \leq_1 z.$
4. $S(x, 0_{D^*}) \in I(x), \ \forall x \in D^*.$

Definition 2.7 *A picture fuzzy t-norm* T *is called representable iff there exist two t-norms* t_1, t_2 *and a t-conorm* s_3 *on [0,1] satisfy:* $T(x,y) = (t_1(x_1,y_1), t_2(x_2,y_2), s_3(x_3,y_3)), \forall x, y \in D^*.$

Definition 2.8 *A picture fuzzy t-conorm* S *is called representable iff there exist two t-norms* t_1, t_2 *and a t-conomrm* s_3 *on [0,1] satisfy:* $S(x,y) = (s_3(x_1,y_1), t_2(x_2,y_2), t_1(x_3,y_3)), \forall x, y \in D^*.$

Some examples: Representable picture fuzzy t-norms, for all $x, y \in D^*$:

1. $T_{\min}(x,y) = (\min(x_1,y_1), \min(x_2,y_2), \max(x_3,y_3)).$
2. $T_2(x,y) = (\min(x_1,y_1), x_2y_2, \max(x_3,y_3)).$
3. $T_3(x,y) = (x_1y_1, x_2y_2, \max(x_3,y_3)).$
4. $T_4(x,y) = (x_1y_1, x_2y_2, x_3 + y_3 - x_3y_3).$
5. $T_5(x,y) = \begin{pmatrix} \begin{cases} x_1 \wedge y_1 & if \ x_1 \vee y_1 = 1 \\ 0 & if \ x_1 \vee y_1 < 1 \end{cases}, \\ \begin{cases} x_2 \wedge y_2 & if \ x_2 \vee y_2 = 1 \\ 0 & if \ x_2 \vee y_2 < 1 \end{cases}, \\ \begin{cases} x_3 \vee y_3 & if \ x_3 \wedge y_3 = 0 \\ 1 & if \ x_3 \wedge y_3 \neq 0 \end{cases} \end{pmatrix}.$
6. $T_6(x,y) = \begin{pmatrix} \max(0, x_1 + y_1 - 1), \\ \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3) \end{pmatrix}.$
7. $T_7(x,y) = \begin{pmatrix} \max(0, x_1 + y_1 - 1), \\ \max(0, x_2 + y_2 - 1), x_3 + y_3 - x_3y_3 \end{pmatrix}.$
8. $T_8(x,y) = \begin{pmatrix} \max\left\{\frac{1}{2}(x_1 + y_1 - 1 + x_1y_1), 0\right\}, \\ \max\left\{\frac{1}{2}(x_2 + y_2 - 1 + x_2y_2), 0\right\}, \\ x_3 + y_3 - x_3y_3 \end{pmatrix}.$
9. $T_9(x,y) = \begin{pmatrix} x_1y_1, \max(0, x_2 + y_2 - 1), \\ x_3 + y_3 - x_3y_3 \end{pmatrix}.$
10. $T_{10}(x,y) = \begin{pmatrix} \max(0, x_1 + y_1 - 1), \\ x_2y_2, x_3 + y_3 - x_3y_3 \end{pmatrix}.$

Some examples: Representable picture fuzzy t-conorms, for all $x, y \in D^*$:

1. $S_{\max}(x,y) = (\max(x_1,y_1), \min(x_2,y_2), \min(x_3,y_3))$.
2. $S_2(x,y) = (\max(x_1,y_1), x_2y_2, \min(x_3,y_3))$.
3. $S_3(x,y) = (\max(x_1,y_1), x_2y_2, x_3y_3)$.
4. $S_4(x,y) = (x_1 + y_1 - x_1y_1, x_2y_2, x_3y_3)$.
5. $S_5(x,y) = \begin{pmatrix} x_1 \vee y_1, \\ \begin{cases} x_2 \wedge y_2 & if\ x_2 \vee y_2 = 1 \\ 0 & if\ x_2 \vee y_2 < 1 \end{cases}, x_3 \wedge y_3 \end{pmatrix}$.
6. $S_6(x,y) = \begin{pmatrix} \begin{cases} x_1 \vee y_1 & if\ x_1 \wedge y_1 = 0 \\ 1 & if\ x_1 \wedge y_1 \neq 0 \end{cases}, \\ x_2 \wedge y_2, \begin{cases} x_3 \wedge y_3 & if\ x_3 \vee y_3 = 1 \\ 0 & if\ x_3 \vee y_3 < 1 \end{cases} \end{pmatrix}$.

III. SOME CLASSES OF IMPLICATION OPERATORS FOR PICTURE FUZZY SETS

In this section, we present some classes of implication operators for picture fuzzy sets, which are the direct generalizations of the classical implication operators and some classes of the fuzzy implication operators (see, for example [3,4]).

First important class of picture implication operators are the followings.
Let $a, b \in D^*$, $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$.

Definition 3.1 *A mapping* $I : D^* \times D^* \to D^*$, *is a picture implication operator of the class 1 if it satisfies the following boundary conditions:*

$$I(0_{D^*}, 1_{D^*}) = 1_{D^*}, where\ 0_{D^*} = (0,0,1), 1_{D^*} = (1,0,0). \tag{3.1}$$
$$I(0_{D^*}, 0_{D^*}) = 1_{D^*}, \tag{3.2}$$
$$I(1_{D^*}, 1_{D^*}) = 1_{D^*}, \tag{3.3}$$
$$I(1_{D^*}, 0_{D^*}) = 0_{D^*} . \tag{3.4}$$

Clearly this definition of picture implication operators is a direct generalization of the classical implication and the definition of fuzzy implication operators given in [3, p. 141].

Another class of the picture implication operators is defined in the following:

Definition 3.2 *A mapping* $I : D^* \times D^* \to D^*$, *is a picture implication operator of the class 2 if it satisfies the following boundary conditions: (3.1) - (3.4) and*

$$I(a_1, b) \geq_1 I(a_2, b), \quad \forall\ a_1 \leq_1 a_2, b \in D^* \tag{3.5}$$
$$I(a, b_1) \leq_1 I(a, b_2), \quad \forall\ b_1 \leq_1 b_2, a \in D^* \tag{3.6}$$

Now we give some direct generalizations of the fuzzy implication operators.

Definition 3.3 *Let* $n(x)$ *be a picture negation operator and let* $S(x, y)$ *be a picture fuzzy t-conorm operator. A mapping* $I : D^* \times D^* \to D^*$, *given by:*

$$I(a,b) = S(n(a), b), \quad \forall a, b \in D^* \tag{3.7}$$

It is a new direct generalization of the fuzzy implications given in the definition 6.1.3 [3, p.146].

Now we give some picture fuzzy implication operators, which are usually referred to in the literature as *S-implications*.

Definition 3.4 *Let* $n(x)$ *be a picture negation operator and let* $S_{max}(x, y)$ *be a picture t-conorm operator. A mapping* $I : D^* \times D^* \to D^*$, *given by:*

$$I(x, y) = S_{\max}(n(x), y), \quad \forall x, y \in D^* \tag{3.8}$$

For example:
For $a,b\in D^*$, $a=(a_1,a_2,a_3)$, $b=(b_1,b_2,b_3)$.
Now we obtain new picture implication operators. Since

$\min(a,b)=(a_1\wedge b_1,a_2\wedge b_2,a_3\vee b_3),$
$\max(a,b)=(a_1\vee b_1,a_2\wedge b_2,a_3\wedge b_3),$ *and*
$n(a)=(a_3,a_4,a_1),$
where $a_4=1-(a_1+a_2+a_3)$

We have

$$I(a,b)=S_{\max}(n(a),b)=S_{max}((a_3,a_4,a_1),(b_1,b_2,b_3))=(a_3\vee b_1,a_4\wedge b_2,a_1\wedge b_3),\quad \forall a,b\in D^* \tag{3.9}$$

If we use $n_0(a)=(a_3,0,a_1)$, we obtain

$$I(a,b)=S_{\max}(n_0(a),b)=S_{\max}((a_3,0,a_1),(b_1,b_2,b_3))=(a_3\vee b_1,0,a_1\wedge b_3),\quad \forall a,b\in D^* \tag{3.10}$$

Picture fuzzy implication operators defined in (3.9) or (3.10) are generalizations of the Kleene-Dienes implication

$I_b(x,y)=\max(1-x,y),$ *where* $x,y\in[0,1],$ in the fuzzy logic.

Definition 3.5 *Let* $n(x)$ *be a picture negation operator and let* $S_{\max}(x,y)$ *be a picture t-conorm operator. Let* $T(x,y)$ *be a picture t-norm operator. A mapping* $I: D^*\times D^*\to D^*$, *given by:*

$$I(x,y)=S_{\max}(n(x),T(x,y)),\quad \forall x,y\in D^* \tag{3.11}$$

Picture fuzzy implication operators defined in (3.11) is are generalizations of the fuzzy Kleene-Dienes implication and the picture implication operator given in the definition 3.6.

For example: now we obtain new picture implication operators. If we choose
$T(a,b)=\min(a,b),$
$\min(a,b)=(a_1\wedge b_1,a_2\wedge b_2,a_3\vee b_3),$
$\max(a,b)=(a_1\vee b_1,a_2\wedge b_2,a_3\wedge b_3),$ *and*
$n(a)=(a_3,a_4,a_1),$
where $a_4=1-(a_1+a_2+a_3)$

We have

$$I(a,b)=S_{\max}(n(a),T(a,b)=S_{max}((a_3,a_4,a_1),\min(a,b)) \tag{3.12}$$
$$=(a_3\vee(a_1\wedge b_1),a_4\wedge(a_2\wedge b_2),a_1\wedge(a_3\vee b_3)),\quad \forall a,b\in D^*$$

Definition 3.6 *A mapping* $I: D^*\times D^*\to D^*$, *given by:*

$$r=\begin{cases}1_{D^*} & \text{if } a<_1 1_{D^*} \text{ or } b=1_{D^*},\\ 0_{D^*} & \text{otherwise}\end{cases}$$

where $r=I(a,b)\in D^*$, $a\in D^*, b\in D^*$,

It is a direct generalization of the standard sharp classical implication operator.

Definition 3.7 *A mapping* $I: D^*\times D^*\to D^*$, *given by:*

$$r=\begin{cases}1_{D^*} & \text{if } a\le_1 b,\\ 0_{D^*} & \text{otherwise}\end{cases}$$

where $r=I(a,b)\in D^*$, $a\in D^*, b\in D^*$,

It is a direct generalization of the standard strict implication operator.
Another picture implication operator is the following:

Definition 3.8 *A mapping* $I : D^* \times D^* \to D^*$*, given by:*

$$r = \begin{cases} 1_{D^*} & if\ a \leq_1 b, \\ b & otherwise \end{cases}$$

where $r = I(a,b) \in D^*,\ a \in D^*, b \in D^*,$

The proof is direct from the definition 3.8.

Definition 3.9 Let n (a) is a picture negation. A mapping $I : D^* \times D^* \to D^*$, given by:

$$r = \begin{cases} 1_{D^*} & if\ a \leq_1 b, \\ max(n(a),b) & otherwise \end{cases}$$

where $r = I(a,b) \in D^*,\ a \in D^*, b \in D^*,$
tt is a new direct generalization of the Fodor's the fuzzy implication and is also an picture implication operator of the class 2.

Definition 3.10 *Let* $n(x)$ *be a picture negation operator and let* $S(x, y)$ *be a picture t-conorm operator. Let* $T(x, y)$ *be a picture t-norm operator. A mapping* $I : D^* \times D^* \to D^*$*, given by:*

$$I(x, y) = S(n(x), T(x, y)), \quad \forall x, y \in D^*, \tag{3.13}$$

Now we give concrete operators combining with the concrete picture t-norms and picture t-conorms.

IV. THE COMPOSITIONAL RULE OF CHAIN INFERENCE

A. The compositional rule of inference

The compositional rule of inference (see [3]) constitutes an inference rule in approximate reasoning in which it is possible to draw vague conclusions from vague premises.

The mathematical pattern of the generalized modus ponens is follows.

Let X and Y be variables taking values in U and V, respectively. Let A **,** A^* **,** and B be fuzzy subsets of appropriate spaces. From "If X is A then Y is B", and "Y is B^*" can be taken as a logical conclusion.

We consider a relation R, and A is a subset of U, then the image of the projection of A into V is the set $B = \{v \in V : (u,v) \in R \ for\ some\ u \in A\}$.

In terms of indicator functions,

$$B(v) = \vee_{x \in U} \{(A \times V)(u,v) \wedge R(u,v)\} = \vee_{x \in U} \{A(u) \wedge R(u,v)\}$$

This can be written as $B = R \circ A$, where $\circ$ is the composition operator of two sets. When R and A^* are fuzzy subsets of $U \times V$ and V, respectively, the same composition $R \circ A^*$ yields a fuzzy subset of V.

When applying this procedure to the generalized modus ponens schema

$$\begin{array}{l} \text{IF} \quad X \text{ is } A^* \\ \quad \text{THEN} \\ \quad (X,Y) \text{ is } R \\ \hline \quad B^* = R \circ A^*, \end{array} \tag{4.1}$$

where R is a fuzzy relation on $U \times V$ representing the conditional " If X is A then Y is B".

Thus if we define $R(u,v) = (A(u) \Rightarrow B(v))$ where $\Rightarrow$ is a fuzzy implication operator and more generally, the special t-norm $T(x, y) = x \wedge y$ can be replaced by an arbitrary fuzzy t-norm operator $T(u,v)$ in the composition operation among relations, leading to the result of the Compositional Rule of Inference (CRI) [3,6]

$$B^*(v) = \vee_{u \in U} \{T((A(u) \Rightarrow B(v)), A(u))\} \tag{4.2}$$

We can choose concrete t-norm operators and concrete fuzzy implication operators to obtain concrete inference procedures in fuzzy logic.

B. Compositional Rule of Inference in Picture Fuzzy Logic Setting, PFL-CRI

Let X and Y be variables assuming values in U and V. Consider picture fuzzy facts X is A^* and (X,Y) are R, where $A^* \in PFS(U)$, $R \in PFR(U \times V)$ (R is a picture fuzzy relation between U and V).
The PFL-CRI allows us to infer the picture fuzzy fact B.
Expressing this under the form of an inference schema, we get

$$\begin{array}{l} \text{If } X \text{ is } A^* \text{ and } (X,Y) \text{ is } R \\ \quad\quad \text{then } Y \text{ is } B = R \circ A^* \end{array} \quad (4.3)$$

We use a picture fuzzy implication operator $I(a,b)$ to define the picture fuzzy relation R. Given picture fuzzy sets $A \in PFS(U)$ and $B \in PFS(V)$, we calculate,

$$(\mu_R(u,v), \eta_R(u,v), \nu_R(u,v)) =$$
$$I((\mu_A(u,v), \eta_A(u,v), \nu_A(u,v)), (\mu_B(u,v), \eta_B(u,v), \nu_B(u,v)))$$
for every $(u,v) \in U \times V$, (4.4)

Thus, we defined the picture fuzzy relation R. Using this definition with the picture fuzzy composition operators of picture fuzzy relations given in [8], it is clear that the PFL- ICR is an extension of the fuzzy-based CRI [5].

Use (4.2) and (4.3) with choosing concrete picture fuzzy t-norms, picture fuzzy implication operators combining with a concrete picture composition operator, (which was given in [8] we obtain the conclusions of the PFl-ICR.

C. Compositional Rule of Chain Inference

Model 1a. From practical applications inference procedures in many cases we have make some steps of inference processes. We have to add information and conditions for computing procedures.

In the model 1a. the first obtained conclusion $B_1 = R \circ A^*$ combining with the new fact $X_2 = A^{**}$ and $R_2(B^* \wedge X_2, Y)$ with the real factor of $(X = A^*, X_2 = A^{**})$ to inference by the composition rule

$$B = (A^*, A^{**}) \circ R_2(B_1^* \wedge X_2, Y) \quad (4.4)$$

V. CONCLUSION

The interest for Picture Fuzzy Sets from the perspective of logical deduction will be continuing to grow. In this paper, we present some classes of implication operators of picture fuzzy logic and a compositional rule of chai inference in a picture fuzzy logic setting. Some applications of the inference procedures were given in [14-16] and some new applications of the new fuzzy theory could be found in [12,13]. We present firstly the compositional rule of chain inference, giving a class of intelligent inference schema for complex computational intelligence problems. In next paper, we will develop some concrete inference procedures, (first ones for some computational intelligence problems were given in [11]), for applying to computing in intelligent systems.

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LUẬT HỢP THÀNH CHO CÁC SUY DIỄN DÃY TRONG CÁC BÀI TOÁN TRÍ TUỆ TÍNH TOÁN

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TÓM TẮT: *Quy tắc hợp thành trong suy diễn dành cho các bài toán trí tuệ tính toán với các thông tin mờ, bất định đóng một vai trò quan trọng . Cần phát triển tính toán , lập luận như thế nào trong những tình hình mới khó khăn hơn ? Một hướng giải quyết là sữ dụng lý thuyết các tập mờ bức tranh. Trong bài báo này sau khi trình bày một số lớp toán tử kéo theo trong logic mờ bức tranh chúng tôi trình bày sơ đồ cơ bản cho quy tắc hợp thành dành cho các suy diễn dãy – một lược đồ sẽ rất có ích cho nhiều lớp bài toán của trí tuệ tính toán.*