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DISTANCE AND DISSIMILARITY MEASURE OF PICTURE FUZZY SETS

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ABSTRACT: *To measure the difference of two fuzzy sets/intuitionistic sets, we can use the distance measure and dissimilarity measure between fuzzy sets. Characterization of distance/dissimilarity measure between fuzzy sets/intuitionistic fuzzy set is important as it has application in different areas: pattern recognition, image segmentation, and decision making. Picture fuzzy set is a generalization of fuzzy set and intuitionistic set, so that it have many application. In this paper, we introduce concepts: difference between PFS-sets, distance measure and dissimilarity measure between picture fuzzy sets. We also provide the formulas for determining these values.*

Keywords*: Picture fuzzy set, difference between PFS-sets, distance measure and dissimilarity measure between picture fuzzy sets.*

I. INTRODUCTION

In many practical problems, we need to compare two objects. Therefore, the question of the process and the way to compare those objects is important. There are some models to measure difference between objects, as a general axiomatic framework for the comparison of fuzzy set. (Bouchon et al. [1]). Fuzzy set and intuitionistic fuzzy set have been used a lot in practical math problems [6, 8, 9, 11]. Distance measure between fuzzy sets and intuitionistic fuzzy sets is also important for many practical applications (Ejegwa et al. [4], Hatzimichailidis et al. [6], Lindblad et al. [8], Muthukumar et al. [12]). Besides, dissimilarity measure between fuzzy sets/intuitionistic fuzzy set is also studied and applied in various matters (Li [7], Faghihi [5], Nguyen [13], Mahmood [10]).

In 2014, Cuong introduced the concept of the picture fuzzy set (PFS-sets) [2], in which a given set is represented by three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership. After that, Son gave the applications of the picture fuzzy set in clustering problems in [15,16,17]. Nguyen et al. [14] use picture fuzzy sets to applied for Geographic Data Clustering. Van Dinh et al. [18] introduce the picture fuzzy set database. Cuong and Hai [3] studied some fuzzy logic operators for picture fuzzy sets. But, difference between PFS-sets and dissimilarity between picture fuzzy sets (the concepts are important in application of picture fuzzy sets) are not yet been research.

In this paper, we introduce the concept difference between PFS-sets, distance measure operators and dissimilarity measure operators between picture fuzzy sets. The rest of paper, in section II, we recall the concept of picture fuzzy set and we introduce the new concept difference between PFS-sets. The function of distance measure between PFS-sets is defined in section III. After, we introduce the function of dissimilarity measure between PFS-sets in section IV. We also illustrate with numerical examples the above measures in decision making in section V.

II. BASIC NOTIONS

Definition 1. Picture fuzzy set (PFS):

$$A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U\}$$

where μ_A is a positive membership function, η_A is neural membership function, γ_A is negative membership of A, $\mu_A(u), \eta_A(u), \gamma_A(u) \in [0,1]$ and $0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1$, for all $u \in U$.

We denote $PFS(U)$ is a collection of picture fuzzy set on U. In which:

$$U = \{(u, 1,0,0) | u \in U\}$$

and $\emptyset = \{(u, 0,0,1) | u \in U\}$.

For $A, B \in PFS(U)$.

- Union of A and B:

$$A \cup B = \{(u, \mu_{A\cup B}(u), \eta_{A\cup B}(u), \gamma_{A\cup B}(u)) | u \in U\}$$

Where $\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\}, \eta_{A\cup B}(u) = \min\{\eta_A(u), \eta_B(u)\}, \gamma_{A\cup B}(u) = \min\{\gamma_A(u), \gamma_B(u)\}$.

- Intersection of A and B:

$$A \cap B = \{(u, \mu_{A\cap B}(u), \eta_{A\cap B}(u), \gamma_{A\cap B}(u)) | u \in U\}$$

Where $\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\}, \eta_{A\cap B}(u) = \min\{\eta_A(u), \eta_B(u)\}, \gamma_{A\cap B}(u) = \max\{\gamma_A(u), \gamma_B(u)\}$.

- Subset: $A \subset B$ iff $\mu_A(u) \le \mu_B(u), \eta_A(u) \le \eta_B(u)$ and $\gamma_A(u) \ge \gamma_B(u)$.
  Now, we define an operator called difference between picture fuzzy sets.

Definition 2. An operator $- : PFS(U) \times PFS(U) \to PFS(U)$ is a difference between PFS-sets if it satisfies follow properties:

(D1) $A \subset B$ iff $A - B = \emptyset$,

(D2) If $B \subset C$ then $B - A \subset C - A$,

(D3) $(A \cap C) - (B \cap C) \subset A - B$,

(D4) $(A \cup C) - (B \cup C) \subset A - B$,

For all $A, B, C \in PFS(U)$.

Theorem 1. The function $- : PFS(U) \times PFS(U) \to PFS(U)$ given by:

$$A - B = \{(u, \mu_{A-B}(u), \eta_{A-B}(u), \gamma_{A-B}(u)) | u \in U\}, \text{ where:}$$

$$\mu_{A-B}(u) = \max\{(0, \mu_A(u) - \mu_B(u)\},$$

$$\eta_{A-B}(u) = \max\{0, \eta_A(u) - \eta_B(u)\} \text{ and,}$$

$$\gamma_{A-B}(u) = \begin{cases} 1 - \mu_{A-B}(u) - \eta_{A-B}(u) \ if \ \gamma_A(u) > \gamma_B(u) \\ \min\{1 + \gamma_A(u) - \gamma_B(u), 1 - \mu_{A-B}(u) - \eta_{A-B}(u)\} \quad \text{if} \quad \gamma_A(u) \le \gamma_B(u) \end{cases}$$

is a difference between PFS-sets.

Proof.

It is easy to see that $0 \le \mu_{A-B}(u) + \eta_{A-B}(u) + \gamma_{A-B}(u) \le 1$, for all $u \in U$.

We verify all condition in definition 2:

- With condition (D1).

+ $A \subset B \Longrightarrow A - B = \emptyset$ is obvious.

+ If $A - B = \emptyset$ then $\mu_{A-B}(u) = \max(0, \mu_A(u) - \mu_B(u)\} = \eta_{A-B}(u) = \max\{0, \eta_A(u) - \eta_B(u)\} = 0$ so that $\mu_A(u) \le \mu_B(u)$ and $\eta_A(u) \le \eta_B(u)$; Hence $\gamma_{A-B}(u) = 1$ so that $\gamma_A(u) \ge \gamma_B(u)$. It means that $A \subset B$.

- With condition (D2).

With $B \subset C$, we have $\mu_B(u) \le \mu_C(u), \eta_B(u) \le \eta_C(u)$ and $\gamma_B(u) \ge \gamma_A(u)$. So that:

+ $\mu_{B-A}(u) = \max(0, \mu_B(u) - \mu_A(u)\} \le \max(0, \mu_C(u) - \mu_A(u)\} = \mu_{C-A}(u)$

+ $\eta_{B-A}(u) = \max(0, \eta_B(u) - \eta_A(u)\} \le \max(0, \eta_C(u) - \eta_A(u)\} = \eta_{C-A}(u)$

+ With negative membership function, we consider some cases:

If $\gamma_A(u) \le \gamma_C(u) \le \gamma_B(u)$ then $\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u) \ge 1 - \mu_{C-A}(u) - \eta_{C-A}(u) = \gamma_{C-A}(u)$.

If $\gamma_C(u) \le \gamma_A(u) \le \gamma_B(u)$ then $\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u) \ge 1 - \mu_{C-A}(u) - \eta_{C-A}(u)$. So that $\gamma_{B-A}(u) \ge min\{1 + \gamma_A(u) - \gamma_C(u), 1 - \mu_{C-A}(u) - \eta_{C-A}(u)\} = \gamma_{C-A}(u)$.

If $\gamma_C(u) \le \gamma_B(u) \le \gamma_A(u)$ then $\gamma_{B-A}(u) = 1 - \mu_{B-A}(u) - \eta_{B-A}(u) \ge 1 - \mu_{C-A}(u) - \eta_{C-A}(u)$ and $\gamma_B(u) - \gamma_A(u) \ge \gamma_C(u) - \gamma_A(u)$. So that $\gamma_{B-A}(u) = \min\{1 + \gamma_A(u) - \gamma_B(u), 1 - \mu_{A-B}(u) - \eta_{A-B}(u)\} \ge min\{1 + \gamma_A(u) - \gamma_C(u), 1 - \mu_{C-A}(u) - \eta_{C-A}(u)\} = \gamma_{C-A}(u)$.

- Similarity, it is possible to show that conditions (D3) and (D4) are also satisfied.

Example 1. Given $U = \{u_1, u_2, u_3\}$ and two PFS-sets: $A = \{(u_1, 0.7, 0.2, 0.1), (u_2, 0.6, 0.1, 0.1), (u_3, 0.6, 0.1, 0.2)\}$, $B = \{(u_1, 0.6, 0.3, 0.1), (u_2, 0.7, 0.05, 0.2), (u_3, 0.4, 0.4, 0.1)\}$. Then, computing by theorem 1, we have:

$$A - B = \{(u_1, 0.1, 0, 0.9), (u_2, 0, 0.05, 0.9), (u_3, 0.2, 0, 0.8).$$

III. DISTANCE MEASURE OF PICTURE FUZZY SETS

In this section, we define the distance measure between picture fuzzy sets.

Definition 3. A function $D: PFS(U) \times PFS(U) \to [0, +\infty)$ is a distance measure between PFS-sets if it satisfies follow properties

(i) PF-dist 1: $D(A,B) = 0$ iff $A = B$,
(ii) PF-dist 2: $D(A,B) = D(B,A)$, for all $A,B \in PFS(U)$,
(iii) PF-dist 3: $D(A,C) \le D(A,B) + D(B,C)$, for all $A,B,C \in PFS(U)$.

There are many formulas that determine the distance between two picture fuzzy sets.

Theorem 2. Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For $A,B \in PFS(U)$. We have some distance measure between picture fuzzy sets

a) $D_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|]$

b) $D_E(A,B) = \{\sum_{i=1}^{n}[(\mu_A(u_i) - \mu_B(u_i))^2 + (\eta_A(u_i) - \eta_B(u_i))^2 + (\gamma_A(u_i) - \gamma_B(u_i))^2]\}^{\frac{1}{2}}$

c) $D_H^m(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_A(u_i) - \mu_B(u_i)|, |\eta_A(u_i) - \eta_B(u_i)|, |\gamma_A(u_i) - \gamma_B(u_i)|\}$

d) $D_E^m(A,B) = \{\sum_{i=1}^{n} \max\{(\mu_A(u_i) - \mu_B(u_i))^2, (\eta_A(u_i) - \eta_B(u_i))^2, (\gamma_A(u_i) - \gamma_B(u_i))^2\}\}^{\frac{1}{2}}$

We easy to verify that the functions in theorem 2 are satisfies properties of distance measure between picture fuzzy sets (def. 3). In there, $D_E(A,B)$ is usually used to measure the distance of objects in geometry, $D_H(A,B)$ is used in the information theory.

IV. DISSIMILARITY MEASURE OF PICTURE FUZZY SETS

In this section, we introduce the concept of dissimilarity for picture fuzzy sets.

Definition 4. A function $DM : PFS(U) \times PFS(U) \to R$ is a dissimilarity measure between PFS-sets if it satisfies follow properties:

(i) PF-Diss 1: $DM(A,B) = DM(B,A)$
(ii) PF-Diss 2: $DM(A,A) = 0$.
(iii) PF-Diss 3: If $A \subset B \subset C$ then $DM(A,C) \ge \max\{DM(A,B), DM(B,C)\}$.
For all $A,B,C \in PFS(U)$.

Theorem 3. Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For any $A,B \in PFS(U)$, a function $DM : PFS(U) \times PFS(U) \to R$ is defined by:

$$DM_C(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|S_A(u_i) - S_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)|]$$

where $S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|$ and $S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$ is a dissimilarity measure between PFS-sets.

Proof.

We check that DM_C satisfies the conditions of definition 3. Indeed, we have:

PF-Diss 1 and PF-Diss 2 are obviously.

With PF-Diss 3, if $A \subset B \subset C$ we have

$$\begin{cases} \mu_A(u_i) \le \mu_B(u_i) \le \mu_C(u_i) \\ \eta_A(u_i) \le \eta_B(u_i) \le \eta_C(u_i) \\ \gamma_A(u_i) \ge \gamma_B(u_i) \ge \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$. So that:

$S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)| \ge S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)| \ge S_C(u_i) = |\mu_C(u_i) - \gamma_C(u_i)|$, and

$|\eta_A(u_i) - \eta_C(u_i)| \ge \max\{|\eta_A(u_i) - \eta_B(u_i)|, |\eta_B(u_i) - \eta_C(u_i)|\}$.

Hence, $DM_C(A,C) \ge max\{DM_C(A,B), DM_C(B,C)\}$. It means PF-Diss 3 satisfy.

We have some dissimilarity measure in theorem 3, as follows.

Theorem 4. Given $U = \{u_1, u_2, \dots, u_n\}$ is an universe set. For any $A,B \in PFS(U)$. We have:

a) $DM_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|]$

b) $DM_L(A,B) = \frac{1}{5n}\sum_{i=1}^{n}[|S_A(u_i) - S_B(u_i)| + |\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|]$

c) $DM_O(A,B) = \frac{1}{\sqrt{3n}}\sum_{i=1}^{n}[(\mu_A(u_i) - \mu_B(u_i))^2 + (\eta_A(u_i) - \eta_B(u_i))^2 + (\gamma_A(u_i) - \gamma_B(u_i))^2]^{\frac{1}{2}}$ are the dissimilarity measure between picture fuzzy sets.

The proof of this theorem is similar to the theorem 3.

V. NUMERICAL EXAMPLES

In this section, we will give some examples using distance and dissimilarity measure in decision making.

Example 2. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows:

$\mathrm{A_1} = \{(\mathrm{u_1}, 0.1, 0.1, 0.1), (\mathrm{u_2}, 0.1, 0.4, 0.3), (\mathrm{u_3}, 0.1, 0, 0.9)\}$,

$\mathrm{A_2} = \{(\mathrm{u_1}, 0.7, 0.1, 0.2), (\mathrm{u_2}, 0.1, 0.1, 0.8), (\mathrm{u_3}, 0.1, 0.1, 0.7)\}$.

Now, there is a sample $\mathrm{B} = \{(\mathrm{u_1}, 0.4, 0, 0.4), (\mathrm{u_2}, 0.6, 0.1, 0.2), (\mathrm{u_3}, 0.1, 0.1, 0.8)\}$

Question: which pattern does B belong to?

+ Applying the distant measure $D_H(A,B)$ we have:

$$D_H(A_1,B) = D_H(A_2,B) = 0.2$$

+ Applying the dissimilarity measure $DM_L(A,B)$ we have:

$$DM_L(A_1,B) = \frac{2.1}{15} < DM_L(A_2,B) = \frac{2.7}{15}.$$

In this example, we see that using the distant measure $D_H(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_1 if we use the dissimilarity measure $DM_L(A,B)$.

Example 3. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$\mathrm{A_1} = \{(\mathrm{u_1}, 0.4, 0.5, 0.1), (\mathrm{u_2}, 0.7, 0.1, 0.1), (\mathrm{u_3}, 0.3, 0.3, 0.2)\}$,

$\mathrm{A_2} = \{(\mathrm{u_1}, 0.5, 0.4, 0), (\mathrm{u_2}, 0.7, 0.2, 0.1), (\mathrm{u_3}, 0.4, 0.3, 0.2)\}$.

$\mathrm{A_3} = \{(\mathrm{u_1}, 0.4, 0.4, 0.1), (\mathrm{u_2}, 0.6, 0.1, 0.1), (\mathrm{u_3}, 0.4, 0.1, 0.4)\}$

Now, there is a sample $\mathrm{B} = \{(\mathrm{u_1}, 0.1, 0.1, 0.6), (\mathrm{u_2}, 0.7, 0.1, 0.2), (\mathrm{u_3}, 0.8, 0.1, 0.1)\}$

Question: which pattern does B belong to?

+ Applying the dissimilarity measure $DM_M(A,B)$ we have:

$$\mathrm{DM_M(A_1,B) = DM_M(A_3,B)} = \frac{2.1}{9};\ \mathrm{DM_M(A_2,B)} = \frac{2.2}{9}.$$

+ Applying the distance measure $D_E(A,B)$ we have:

$$\mathrm{D_E(A_1,B)} = 0.9;\ \mathrm{D_E(A_2,B)} = 0.916515139;\ \mathrm{D_E(A_3,B)} = 0.8366600265$$

In this example, we see that using the dissimilarity measure $DM_H(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_3 if we use the distance measure $D_E(A,B)$.

Example 4. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3, u_4\}$ as follows:

$\mathrm{A_1} = \{(\mathrm{u_1}, 0.3, 0.4, 0.1), (\mathrm{u_2}, 0.3, 0.4, 0.1), (\mathrm{u_3}, 0.6, 0.1, 0.2), (\mathrm{u_4}, 0.6, 0.1, 0.2)\}$,

$\mathrm{A_2} = \{(\mathrm{u_1}, 0.4, 0.4, 0.1), (\mathrm{u_2}, 0.3, 0.2, 0.4), (\mathrm{u_3}, 0.6, 0.1, 0.3), (\mathrm{u_4}, 0.5, 0.2, 0.2)\}$,

$\mathrm{A_3} = \{(\mathrm{u_1}, 0.4, 0.4, 0.1), (\mathrm{u_2}, 0.3, 0.1, 0.3), (\mathrm{u_3}, 0.6, 0.1, 0.2), (\mathrm{u_4}, 0.5, 0.2, 0.1)\}$.

Now, there is a sample

$\mathrm{B} = \{(\mathrm{u_1}, 0.35, 0.65, 0), (\mathrm{u_2}, 0.55, 0.35, 0.1), (\mathrm{u_3}, 0.65, 0.1, 0.1), (\mathrm{u_4}, 0.6, 0.15, 0.2)\}$

Question: *which pattern does B belong to?*

+ Applying the distance measure $D_H^m(A,B)$ we have:

$$\mathrm{D_H^m(A_1,B) = D_H^m(A_3,B)} = 0.7;\ \mathrm{D_H^m(A_2,B)} = 0.85 .$$

+ Applying the dissimilarity measure $DM_C(A,B)$ we have:

$$\mathrm{DM_C(A_1,B)} = 0.0875;\ \mathrm{DM_C(A_2,B) = DM_C(A_3,B)} = 0.1$$

In this example, we see that using the distance measure $D_H^m(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_1 if we use the dissimilarity measure $DM_C(A,B)$.

Example 5. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2\}$ as follows:

$\mathrm{A_1} = \{(\mathrm{u_1}, 0.4, 0.5, 0.1), (\mathrm{u_2}, 0.3, 0.4, 0.2)\}$,

$\mathrm{A_2} = \{(\mathrm{u_1}, 0.5, 0.4, 0.1), (\mathrm{u_2}, 0.4, 0.3, 0.1)\}$.

Now, there is a sample $\mathrm{B} = \{(\mathrm{u_1}, 0.1, 0.1, 0.1), (\mathrm{u_2}, 0.5, 0.5, 0)\}$

Question: *which pattern does* B *belong to?*

+ Applying the distant measure $D_E^m(A,B)$ we have:

$$\mathrm{D_E^m(A_1, B) = D_E^m(A_2, B)} = 0.44721$$

+ Applying the dissimilarity measure $DM_O(A,B)$ we have:

$$\mathrm{DM_O(A_1, B)} = 0.3265;\ \mathrm{DM_O(A_2, B)} = 0.3041241$$

In this example, we see that using the distant measure $D_E^m(A,B)$ can not be used to classify the sample B. But, we can see that B belongs to pattern A_2 if we use the dissimilarity measure $DM_O(A,B)$.

VI. CONCLUSION

In this paper, we introduce the concepts of the difference between PFS-sets, distance measure and dissimilarity between picture fuzzy sets. We give some distant measure and dissimilarity measure of picture fuzzy sets. Beside, we Illustrate with numerical examples the above measures in decision making. In the future, we will study the properties of these measure and applications of them in practical problems.

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ĐỘ ĐO KHOẢNG CÁCH VÀ ĐỘ ĐO KHÔNG TƯƠNG TỰ GIỮA CÁC TẬP MỜ BỨC TRANH

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TÓM TẮT: *Để đánh giá sự khác nhau giữa hai tập mờ (hoặc tập mờ trực cảm), chúng ta có thể đưa ra độ đo khoảng cách hay độ đo sự không tương tự giữa chúng. Đặc trưng về độ đo khoảng cách và độ đo không tương tự giữa hai tập mờ là rất quan trọng vì nó có nhiều ứng dụng trong các lĩnh vực như nhận dạng mẫu, phân tích ảnh và hỗ trợ ra quyết định. Tập mờ bức tranh là sự tổng quát của tập mờ và tập trực mờ trực cảm, vì vậy nó có nhiều ứng dụng. Trong bài báo này, chúng tôi giới thiệu các khái niệm: hiệu hai tập mờ bức tranh; độ đo khoảng cách; độ đo không tương tự giữa các tập mờ bức tranh và đưa ra các công thức để xác định các đại lượng này.*

Từ khóa: *Tập mờ bức bức tranh (PFS), hiệu hai tập mờ PFS, độ đo khoảng cách và độ đo không tương tự của các tập mờ PFS.*